1 The line $3x+2y=12$ meets the curve $x^2-y+2y^2=12$ at the points A and B. Calculate the length of AB. [7]

2 If the difference between the roots of the equation $x^2+px+q=0$ is 3, show that $p^2=4q+9$. [4]

3 The roots of the equation $100x^2-29x+1=0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, such that α and β are positive. [5]

4 The line $y=mx+1$, where m is a constant, intersect the curve $y=x^2-3x+2$ at two distinct points. Find the range of values of m. [5]

5 A piece of wire of length of 24 cm is bent into a rectangle. Let x cm be the length of one side of the rectangle and A cm^2 be the area of the rectangle.

(i) Express A in terms of x. [1]

(ii) Find the range of values of x such that the area of the rectangle is greater than 27 cm^2. [5]

(iii) Hence, find the maximum of the rectangle. [1]

6 Jane threw a ball such that the height, s metres, of the ball at time t seconds is given by the equation $s=-5.1t^2+vt+2.5$, where v m/s is the speed at which she threw the ball. Use the discriminate to determine whether the ball could reach a height of 15 m if it is thrown at speed of 20 m/s. [4]

7 Solve the following equations.

(a) $\sqrt{5x+2}-\sqrt{3x-8}=0$ [2]

(b) $3\sqrt{x-1}=2\sqrt{x+4}$ [2]

(c) $\sqrt{7-6x}+x=-3x$ [4]

8 It is given that x and y are rational numbers. Find the values of x and y in

$(6-3\sqrt{5})(x+y\sqrt{5})=81-30\sqrt{5}$. [7]

9 An open cuboid bin has a square base of side $(\sqrt{7}-\sqrt{5})$ m. The capacity of the bin is $(90\sqrt{5}-76\sqrt{7})$ m^3. Find the exact value of

(a) the base area of the bin, [2]
(b) the height of the bin, [3]
(c) the total surface area of the bin. [3]

10 **Solve the following equations.**

(a) $4^{x+1}+8=33(2^x)$ [5]

(b) $7^{2x+3}\div 7^{x^2}=1$ [4]

11 **(a)** In a lucky draw, $(x-3)$ winners shared a sum of $\$(3x^3-5x^2+6x-54)$ equally. Find the share of each winner. [3]

(b) In a given cubic polynomial $f(x)$, the coefficient of x^3 is 1 and the roots of $f(x)=0$ are -2, 2 and k. When $f(x)$ is divided by $x+1$, the remainder is -6.

(i) Find the value of k. [4]
(ii) Find the remainder when $f(x)$ is divided by x^2-3. [4]

12 Express $\dfrac{5x^2-4x+2}{(3x-4)(x^2+1)}$ in partial fraction. [8]

13 **(a)** Factorise each of the following

(i) $1000a^3-b^3$ [2]
(ii) $3x^4+81x$ [2]

(b) **(i)** Sketch the graph of $y=4e^x$. [2]

(ii) Add the line $y=4+x$ to your graph. [1]

(iii) Hence state the number of solutions of the equation

$4e^x=5+x$. [1]

14 **(a)** **(i)** Expand $\left(1+\frac{x}{4}\right)^9$ up to the first 3 terms. [1]

(ii) Hence, given that $\left(8-2x-3x^2\right)\left(1+\frac{x}{4}\right)^9 = 8+hx+kx^2+.......$,

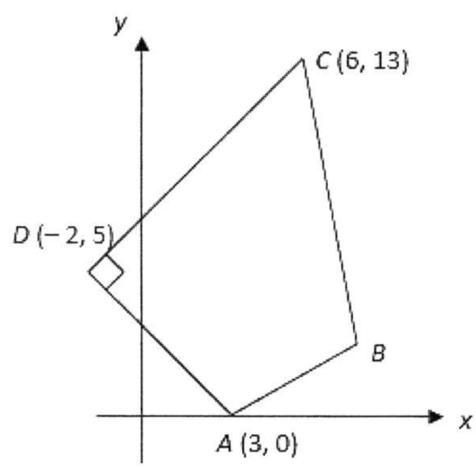
find the values of h and k. [4]

(b) Evaluate the coefficient of x^7 in the binomial expansion of $\left(x^2-\frac{1}{2x}\right)^{14}$. [3]

15 **Solutions to this equation by accurate drawing will not be accepted.**

The diagram shows a quadrilateral $ABCD$ in which $A(3,0)$, $C(6,13)$ and $D(-2,5)$.

The equation of AB is $5y = 3x - 9$ and $\angle ADC = 90^0$.



Find

(i) the equation of AD, [2]

(ii) the perpendicular bisector of CD. [3]

The perpendicular bisector of CD passes through B.

(iii) Find the coordinates of B. [2]

(iv) Find the area of the quadrilateral $ABCD$. [2]

End of Paper

MYE 3E AM 2016 Solution

1 The line $3x + 2y = 12$ meets the curve $x^2 - y + 2y^2 = 12$ at the points *A* and *B*. Calculate the length of *AB*.

Solution

$$x = \frac{12 - 2y}{3}$$

$$\left(\frac{12 - 2y}{3}\right)^2 - y + 2y^2 = 12$$

Some students expressed

$(12 - 2x)^2 = 144 - 4y^2$ **X**

$$\left(\frac{144 - 48y + 4y^2}{9}\right) - y + 2y^2 = 12$$

$$22y^2 - 57y + 36 = 0$$

$$(11y - 12)(2y - 3) = 0$$

$$y = \frac{12}{11},\ y = \frac{3}{2}$$

$$x = 3\frac{3}{11},\ x = 3$$

$$AB = \sqrt{\left(3\frac{3}{11} - 3\right)^2 + \left(\frac{12}{11} - \frac{3}{2}\right)^2} = 0.492 \text{ units}$$

Some students wrote cm instead units. Some left units out.

A lot of students used $(x_1 + x_2)^2$ instead $(x_1 - x_2)^2$. Or some did not square the brackets. **X** **Some did not know the formula.**

2 The roots of the equation $100x^2 - 29x + 1 = 0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, such that α and β are positive.

$$\alpha^2 + \beta^2 = \frac{29}{100}$$

$$(\alpha\beta)^2 = \frac{1}{100}$$

$$\alpha\beta = \frac{1}{10}$$

A lot students did not reject $\alpha\beta = -\frac{1}{10}$

$$(\alpha + \beta)^2 - 2\alpha\beta = \frac{29}{100}$$

$$(\alpha + \beta)^2 - 2\left(\frac{1}{10}\right) = \frac{29}{100}$$

$$(\alpha + \beta) = \frac{7}{10}$$

A lot students did not reject $\alpha + \beta = -\frac{7}{10}$

$$x^2 - \left(\frac{7}{10}\right)x + \frac{1}{10} = 0$$

$$10x^2 - 7x + 1 = 0$$

Some students did not write (= 0) for the required equation.

3 The line $y = mx + 1$, where m is a constant, intersect the curve $y = x^2 - 3x + 2$ at <u>two distinct points</u>. Find the range of values of m.

$$x^2 - 3x + 2 - mx - 1 = 0$$

b = - 3 - m, some student identified b = 3 + m | **X**

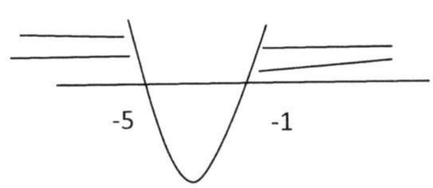
$$(-3 - m)^2 - 4(1)(1) > 0$$

$$9 + 6m + m^2 - 4 > 0$$

$$m^2 + 6m + 5 > 0$$

$$(m + 5)(m + 1) > 0$$

$$m < -5, \quad m > -1$$



4 A piece of wire of length of 24 cm is bent into a rectangle. Let x cm be the length of one side of the rectangle and A cm^2 be the area of the rectangle.

(i) Express A in terms of x.

(ii) Find the range of values of x such that the area of the rectangle is greater than 27 cm^2.

(iii) Hence, find the maximum area of the rectangle.

<u>This question is not well-done.</u>

(i) $A = x(12 - x)$

Some students left this question totally blank. Very few students drew a rectangle to analyse the question

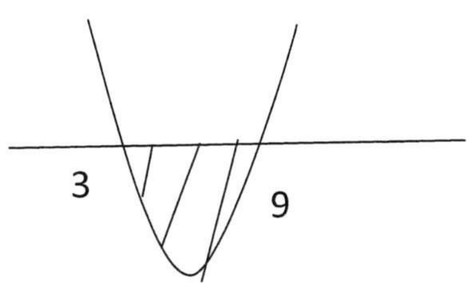
$$\frac{24 - 2x}{2} = 12 - x$$

A simple diagram will help a lot !!!!!!!!!!!

(ii) $x(12 - x) > 27$

$$x^2 - 12x + 27 < 0$$

$$(x - 9)(x - 3) < 0$$



$$3 < x < 9$$

Only a few students were able to use the mid-value to locate the Maximum Area.

(iii) Maximum A occurs at $x = 6$

Maximum A = (12 - 6)(6) = 36 cm^2

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5 Jane threw a ball such that the height, s metres, of the ball at time t seconds is given by the equation $s = -5.1t^2 + vt + 2.5$, where v m/s is the speed at which she threw the ball. Use the discriminant to determine whether the ball could reach a height of 15 m if it is thrown at speed of 20 m/s.

This question is badly done despite the hint to use the discriminate to show the conclusion.

$15 = -5.1t^2 + 20t + 2.5$ **Many did not equate S = 15.**

$20^2 - 4(-5.1)(-12.5) = 145 > 0$

Hence the ball could reach a height of 15 m at 20 m/s.

6 Solve the following equations.

(a) $\sqrt{5x+2} - \sqrt{3x+8} = 0$

(b) $\sqrt{7-6x} + x = -3x$

Solution

(a) $\left(\sqrt{5x+2}\right)^2 = \left(\sqrt{3x+8}\right)^2$

$5x + 2 = 3x + 8$

$x = 3$

This was well-done.

(b)

$$7-6x=16x^2$$

$$16x^2+6x-7=0$$

$$(2x-1)(8x+7)=0$$

$$x=\frac{1}{2}, \quad x=-\frac{7}{8}$$

(rejected)

Many students did check the feasibility of x = ½ thus many did not reject x = ½ .

7 It is given that *x* and *y* are rational numbers. Find the values of x and y in

$$(6-3\sqrt{5})(x+y\sqrt{5})=81-30\sqrt{5}.$$

Solution

$$6x+6y\sqrt{5}-3x\sqrt{5}-3y(5)=81-30\sqrt{5}$$

$$6x-15y+\sqrt{5}(6y-3x)=81-30\sqrt{5}$$

$$6x-15y=81$$

$$6y-3x=-30$$

Solving

2(2y +10) - 5y = 27

$$y=-7$$

$$x=-4$$

This was well-done.

8 **An open cuboid bin has a square base of side $(\sqrt{7}-\sqrt{5})$ m. The capacity of the bin is $(90\sqrt{5}-76\sqrt{7})$ m^3. Find the exact value of**

(a) the base area of the bin,
(b) the height of the bin,
(c) the total surface area of the bin.

Solution

(a) Base area $= (\sqrt{7}-\sqrt{5})^2 = 12-2\sqrt{35}$

(b) Height $= \dfrac{(90\sqrt{5}-76\sqrt{7})}{(12-2\sqrt{35})} \times \dfrac{(12+2\sqrt{35})}{(12+2\sqrt{35})}$

$$= \frac{1080\sqrt{5}+180\sqrt{175}-912\sqrt{7}-152\sqrt{245}}{144-4\times 35}$$

Some students did not or could not simply from the above line to the following line.

$$= \frac{1080\sqrt{5}+180\times 5\sqrt{7}-912\sqrt{7}-1064\sqrt{5}}{4}$$

$$= \frac{16\sqrt{5}-12\sqrt{7}}{4} = 4\sqrt{5}-3\sqrt{7}$$

(c) **Total surface area** $= 12-2\sqrt{35}+4(\sqrt{7}-\sqrt{5})(4\sqrt{5}-3\sqrt{7})$

It is an opened bin so there should not have $2(12-2\sqrt{35})$. X

$$= 12-2\sqrt{35}+28\sqrt{35}-164$$

$$= 26\sqrt{35}-152$$

9 Solve the following equations. (WELL-DONE)

(a) $4^{x+1} + 8 = 33(2^x)$

(b) $7^{2x+3} \div 7^{x^2} = 1$

(a) $2^{2x+2} + 8 = 33(2^x)$

$2^{2x} \times 2^2 + 8 = 33(2^x)$

Let $y = 2^x$

$4y^2 - 33y + 8 = 0$

$(4y-1)(y-8) = 0$

$y = \frac{1}{4}. \quad y = 8$

$x = -2, \quad x = 3$

(b) $7^{2x+3-x^2} = 7^0$

$2x + 3 - x^2 = 0$

$(x-3)(x+1) = 0$

$x = 3, x = -1$

10 **(a) In a lucky draw, $(x-3)$ winners shared a sum of $\$(3x^3 - 5x^2 + 6x - 54)$ equally. Find the share of each winner.**

(b) In a given cubic polynomial $f(x)$, the coefficient of x^3 is 1 and the roots of $f(x) = 0$ are -2, 2 and *k*. When $f(x)$ is divided by $x+1$, the remainder is -6.

(i) Find the value of *k*.

(ii) Find the remainder when $f(x)$ is divided by $x^2 - 3$.

Solution

(a)

	3	-5	6	-54
3	↓	9	12	54
	3	4	18	

Answer $\$(3x^2 + 4x + 18)$

Remark: Some students left this blank.

(b) (i) $f(x) = (x+2)(x-2)(x-k)$

Some students wrote $(x + k)$ instead of $(x - k)$

$f(-1) = -6$

$(-1+2)(-1-2)(-1-k) = -6$

$3 + 3k = -6$

$k = -3$

(ii) $f(x) = x^3 + 3x^2 - 4x - 12$

$$x + 3$$

$$x^2 - 3 \overline{\big)\, x^3 + 3x^2 - 4x - 12}$$

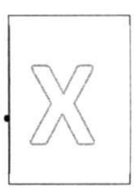
$$-x^3 + 0x^2 + 3x$$

$$3x^2 - x - 12$$

$$-3x^2 \quad + 9$$

$$-x - 3$$

Remainder = $-x - 3$

Some substituted x = $\sqrt{3}$ instead. 

11 **Express** $\dfrac{5x^2 - 4x + 2}{(3x-4)(x^2+1)}$ **in partial fraction.**

Solution

$$\frac{5x^2 - 4x + 2}{(3x-4)(x^2+1)} = \frac{A}{(3x-4)} + \frac{Bx + C}{x^2 + 1}$$

Some students wrote $x^2 + 1 = (x+1)(x-1)$ X

$5x^2 - 4x + 2 \;=\; A(x^2+1) + (3x-4)(Bx+C)$

Let $x = 0$

A = 2 + 4 C--------------------------(1)

Let $x = 1$

3 = 2A – B – C --------------------(2)

Let $x = -1$

11 = 2A + 7B – 7C------------------(3)

Sub A = 2 + 4C into eqns (2) and (3)

7C – B = -1--------------------------(4)

C = 7 – 7B ---------------------------(5)

Solving

A = 2, B = 1, C = 0

Answer $\dfrac{2}{3x-4} + \dfrac{x}{x^2+1}$

Comparing coeffs:

$$5 = A + 3B$$

$$B = \frac{5 - A}{3}$$

$$-4 = -4B + 3C$$

$$C = \frac{A - 2}{4}$$

$$-4 = -4\left(\frac{5 - A}{3}\right) + \left(\frac{3A - 6}{4}\right)$$

$$A = 2, B = 1, \ C = 0$$

12 **(a)** Factorise each of the following

(i) $1000a^3 - b^3$

(ii) $3x^4 + 81x$

(b) **(i)** Sketch the graph of $y = 4e^x$.

(ii) Add the line $y = 4 + x$ to your graph.

(iii) Hence state the number of solutions of the equation

$4e^x = 4 + x$.

(a) (i) $(10a)^3 - b^3$

$= (10a - b)(100a^2 + 10ab + b^2)$

(ii) $3x(x^3 + 3^3)$

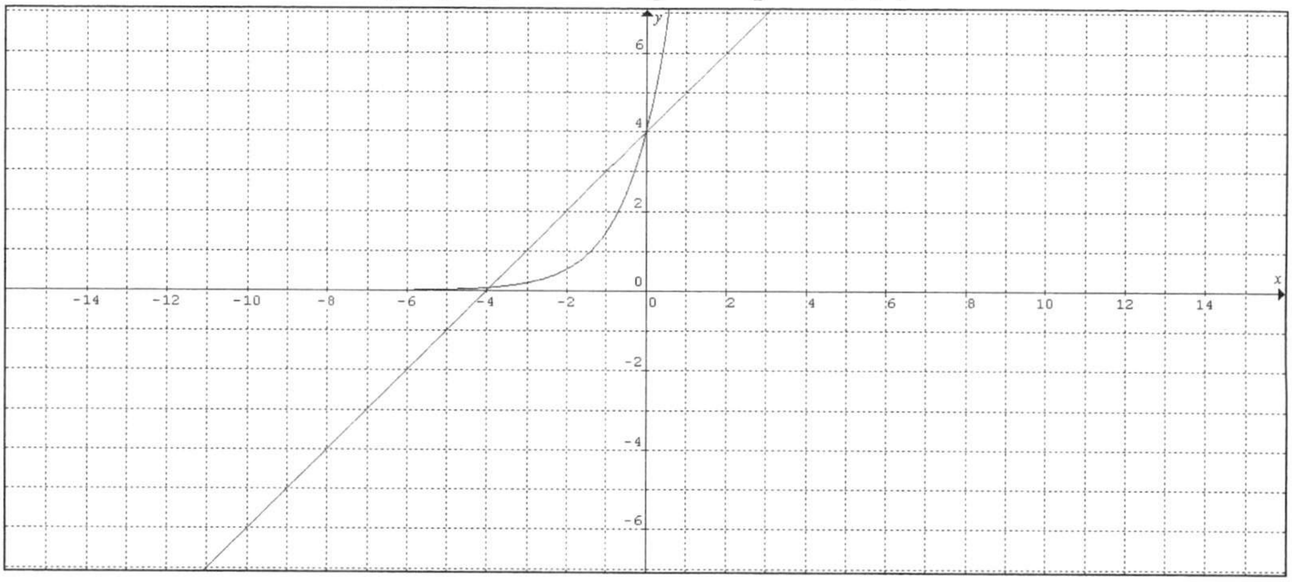
$= 3x(x + 3)(x^2 - 3x + 9)$

12a was badly done.

Many were unable to apply the cubic formulae.

b(i) shape and asymptote of

$y = 4e^x$ **and the graph pass through the point (0,4)**



(ii) Line $y = 4 + x$ (iii) 2 points of intersections

The two graphs were not well-drawn. Many were not able to draw the graphs to occupy both first and second quadrants of the axes. The line did not pass through the two axes.

Therefore many students were not able to obtain the 2 points of intersections.

13 **(a)** **(i)** Expand $\left(1+\frac{x}{4}\right)^9$ up to the first 3 terms.

(ii) Hence, given that $\left(8-2x-3x^2\right)\left(1+\frac{x}{4}\right)^9 = 8 + hx + kx^2 + \ldots\ldots.,$

find the values of h and k.

(b) Evaluate the coefficient of x^7 in the binomial expansion of $\left(x^2 - \frac{1}{2x}\right)^{14}$.

13(a)

(i) $\left(1+\frac{x}{4}\right)^9 = 1 + \frac{9x}{4} + \frac{9x^2}{4} + \ldots\ldots.$ Well-done.

(ii) $\left(8-2x-3x^2\right)\left(1+\frac{9x}{4}+\frac{9x^2}{4}+\ldots\right)$

$= 8+18x+18x^2-2x-\frac{9x^2}{2}-3x^2+...$

$= 8+16x+\frac{21x^2}{2}+...$

$\therefore\ h=16$ and $k=\frac{21}{2}$

Carelessness in expansion resulting in wrong answers.

13(b) $\left(x^2-\frac{1}{2x}\right)^{14}$

General term or $(r+1)^{th}$ term of the expansion

$= {}^{14}C_r\left(x^2\right)^{14-r}\left(-\frac{1}{2x}\right)^r$

$= {}^{14}C_r\left(-\frac{1}{2}\right)^r x^{28-3r}$ [M1]

For the term in x^7, $x^7 = x^{28-3r}$

7 = 28 − 3*r*

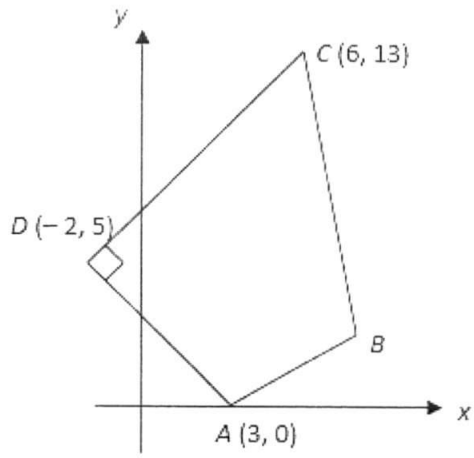
r = 7 [M1]

$\therefore$ Coeff of $x^7 = {}^{14}C_7\left(-\frac{1}{2}\right)^7 = -26\frac{13}{16}$ [A1]

Some students were unable to apply the general formula. They were unable to write down $\left(-\frac{1}{2}\right)^r$ in the working thus unable to obtain the correct answer.

14 Solutions to this equation by accurate drawing will not be accepted. The diagram shows a quadrilateral *ABCD* in which $A(3,0)$, $C(6,13)$ and $D(-2,5)$.

The equation of *AB* is $5y=3x-9$ and $\angle ADC = 90^0$.



Find

(i) the equation of *AD*,

(ii) <u>the perpendicular bisector of *CD*.</u>

The perpendicular bisector of *CD* passes through *B*.

(iii) Find the coordinates of *B*.

(iv) Find the area of the quadrilateral *ABCD*.

14 **(i)** gradient of *AD* = -1

$3 = -1(0) + c$

eq of AD : $y = -x + 3$

(ii) mid-point of CD = (2, 9)

Gradient of perpendicular bisector = -1

Equation of perpendicular bisector : $y - 9 = -(x - 2)$

(iii) $5(-x + 11) = 3x - 9$

$x = 8$

$y = 3$

B (8, 3)

(iv) Area = $\frac{1}{2}\begin{vmatrix} 3 & 8 & 6 & -2 & 3 \\ 0 & 3 & 13 & 5 & 0 \end{vmatrix} = 68 \text{ units}^2$

This question was well-done. Some students think that the point (2,9) is the perpendicular bisector.