Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL

SEMESTRAL EXAMINATION ONE

SECONDARY THREE

ADDITIONAL MATHEMATICS **11 May 2016**

2 hours

Additional Materials:
Writing paper (5 sheets)

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

---

1. (a) Simplify $\sqrt{80} + \sqrt{180} - \dfrac{8}{\sqrt{5}+1}$. [3]

   (b) Simplify $\dfrac{2^{n+1} + 2^n}{4^{\frac{1}{2}n-1} - 2^{n-3}}$. [3]

2. (a) Solve the equation $3^{2x+1} = 10$. [2]

   (b) Given that $\log_2 x^3 = m$ and $\log_4 y = n$, find $\log_2 \sqrt{xy}$ in terms of m and n. [4]

3. Solve the equations

   (a) $2^x\left(4^{x-1}\right) = 8^{2x-1}$, [3]

   (b) $\log_5 x - \log_{25}(x+10) = \dfrac{1}{2}$. [4]

4. Sketch the graph of $y = \ln x$. Insert in your sketch an additional graph required to illustrate how a graphical solution of the equation $xe^{3x} = 1$ may be obtained. State the equation of the additional graph and the number of solutions to the equation $xe^{3x} = 1$. [6]

5. (a) Given that $\sin 110° = p$, express each of the following in terms of p.

   (i) $\sin 70°$ (ii) $\tan 20°$ [4]

   (b) Without the use of a calculator, find the exact value of $\sin\left[\cos^{-1}\left(-\dfrac{2}{3}\right)\right]$. [2]

6. Given that $\tan A = -\dfrac{5}{12}$ and that $\tan A$ and $\cos A$ have opposite signs, find the value of

   (i) $\sin(-A)$ (ii) $\sin(90° - A)$ [4]

7. The value, V dollars, of an antique is given by $V = V_0 e^{kt}$, where V_0 dollars is the initial value of the antique when it was produced, t is the time in years since it was produced and k is a constant.

(i) Find the value of k given that the value of the antique doubled after 7 years. [2]

(ii) Given further that the antique was produced in 1930 and that the value of the antique is evaluated at the beginning of every year, find the year in which its value first exceeded ten times the initial value. [3]

8. The curve $y = a\cos bx + c$, where a and b are positive integers, has an amplitude of 3 and a period of $180°$. The maximum value of y is 1.

(i) State the values of a, b and c. [3]

(ii) With the values stated in part (i), sketch the curve of $y = a\cos bx + c$ for $0° \le x \le 360°$. [3]

9. The roots of $x^2 + 3x - 6 = 0$ are α and β. The roots of another equation $x^2 - 6x + q = 0$ are $\frac{n}{\alpha^3}$ and $\frac{n}{\beta^3}$, where n and q are constants. Find the value of n and of q. [6]

10. Given that $f(x) = mx^3 - (5m-1)x^2 + (m+1)x + m^2$ is exactly divisible by $x-1$ but not by $x-4$,

(i) show that $m = 1$, [4]

(ii) using the value of m shown in part (i), solve the equation $f(x) = 0$, giving your answers correct to two decimal places where necessary. [4]

11. (a) Find the values of k if the graph of $y = (2k-1)x^2 + 2k + 4$ and the line $y = 3kx$ meet at one point only. [4]

(b) Find the range of values of h for which $(h+3)x^2 - 3x > x + h$ for all real values of x. [4]

(c) Show that $2x^2 + p = 2(x-1)$ has no real roots if $p > -\frac{3}{2}$. [3]

12. (i) Given that $2x^3 - 31x - 27 = A(x-4)(x+2)^2 + Bx + C$, find the values of the constants A, B and C. [4]

(ii) Hence or otherwise, express $\frac{2x^3 - 31x - 27}{(x-4)(x+2)^2}$ in partial fractions. [5]

--- End of Paper ---

2016 SECONDARY 3 ADDITIONAL MATHEMATICS SA1
MARKING SCHEME

1. (a)

$$\sqrt{80}+\sqrt{180}-\frac{8}{\sqrt{5}+1}$$

$$=4\sqrt{5}+6\sqrt{5}-\frac{8}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1} \qquad [M1]$$

$$=10\sqrt{5}-\frac{8\left(\sqrt{5}-1\right)}{5-1}$$

$$=10\sqrt{5}-2\left(\sqrt{5}-1\right) \qquad [M1]$$

$$=10\sqrt{5}-2\sqrt{5}+2$$

$$=8\sqrt{5}+2 \quad or \quad 2\left(4\sqrt{5}+1\right) \qquad [A1]$$

(b)

$$\frac{2^{n+1}+2^n}{4^{\frac{1}{2}n-1}-2^{n-3}}$$

$$=\frac{2\left(2^n\right)+2^n}{2^{-2}\left(2^n\right)-2^{-3}\left(2^n\right)} \qquad [M1]$$

$$=\frac{2^n\left(2+1\right)}{2^n\left(\frac{1}{4}-\frac{1}{8}\right)} \qquad [M1]$$

$$=24 \qquad [A1]$$

2. (a)

$$3^{2x+1}=10$$

$$\lg 3^{2x+1}=\lg 10$$

$$(2x+1)\lg 3=1 \qquad [M1]$$

$$2x+1=\frac{1}{\lg 3}$$

$$x=\frac{1}{2}\left(\frac{1}{\lg 3}-1\right)$$

$$x=0.548 \ \ (3s.f.) \qquad [A1]$$

(b) $\log_2 x^3 = m$ $\log_4 y = n$

$3\log_2 x = m$ $\dfrac{\log_2 y}{\log_2 4} = n$

$\log_2 x = \dfrac{1}{3}m$ [M1] $\log_2 y = 2n$ [M1]

$$\log_2 \sqrt{xy} = \frac{1}{2}\log_2 xy$$

$$= \frac{1}{2}(\log_2 x + \log_2 y) \qquad [M1]$$

$$= \frac{1}{2}\left(\frac{1}{3}m + 2n\right)$$

$$= \frac{1}{6}m + n \qquad [A1]$$

3. (a) $2^x\left(4^{x-1}\right) = 8^{2x-1}$

$2^x\left(2^{2x-2}\right) = 2^{6x-3}$ [M1]

$2^{3x-2} = 2^{6x-3}$

By comparing indices,

$3x - 2 = 6x - 3$ [M1]

$3x = 1$

$x = \dfrac{1}{3}$ [A1]

(b) $\log_5 x - \log_{25}(x+10) = \dfrac{1}{2}$

$\log_5 x - \dfrac{\log_5(x+10)}{\log_5 5^2} = \dfrac{1}{2}$ [M1]

$\log_5 x - \dfrac{\log_5(x+10)}{2} = \dfrac{1}{2}$ [M1]

$2\log_5 x - \log_5(x+10) = 1$

$\log_5 \dfrac{x^2}{x+10} = 1$

$\dfrac{x^2}{x+10} = 5$ [M1]

$x^2 = 5x + 50$

$x^2 - 5x - 50 = 0$

$(x-10)(x+5) = 0$

$x = 10 \quad or \quad x = -5\ (rej)$ [A1]

4. $xe^{3x} = 1$

$\ln xe^{3x} = \ln 1$

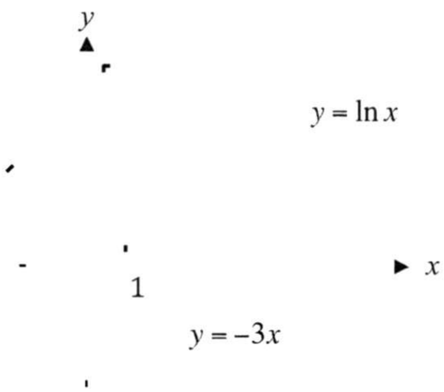
$\ln x + \ln e^{3x} = 0$ [$M1$]

$\ln x + 3x = 0$

$\ln x = -3x$

Equation of additional graph is $y = -3x$. [$A1$]

Number of solution is 1. [A1]



2 marks for graphs (1 mark each). 1 mark for labels and intercepts.

5. (ai) $\sin 70° = \sin(180° - 110°)$ [$M1$]

$= \sin 110°$

$= p$ [$A1$]

(aii) $\tan 20° = \dfrac{\sqrt{1-p^2}}{p}$ [$B2$]

(b) Let $A = \cos^{-1}\left(-\dfrac{2}{3}\right)$.

$\cos A = -\dfrac{2}{3}$ [$M1$]

$\sin A = \dfrac{\sqrt{5}}{3}$ [$A1$]

6. (i) $\sin(-A) = -\sin A$ [$M1$]

$$= -\left(-\frac{5}{13}\right)$$

$$= \frac{5}{13}$$ [$A1$]

(ii) $\sin(90° - A) = \cos A$ [$M1$]

$$= \frac{12}{13}$$ [$A1$]

7. (i) When $V = 2V_0$ and $t = 7$,

$2V_0 = V_0 e^{7k}$ [$M1$]

$e^{7k} = 2$

$k = \frac{1}{7}\ln 2$

$k = 0.0990$ $(3s.f.)$ [$A1$]

(ii) When $V = 10V_0$ and $k = \frac{1}{7}\ln 2$,

$V_0 e^{\left(\frac{1}{7}\ln 2\right)t} > 10V_0$ [$M1$]

$e^{\left(\frac{1}{7}\ln 2\right)t} > 10$

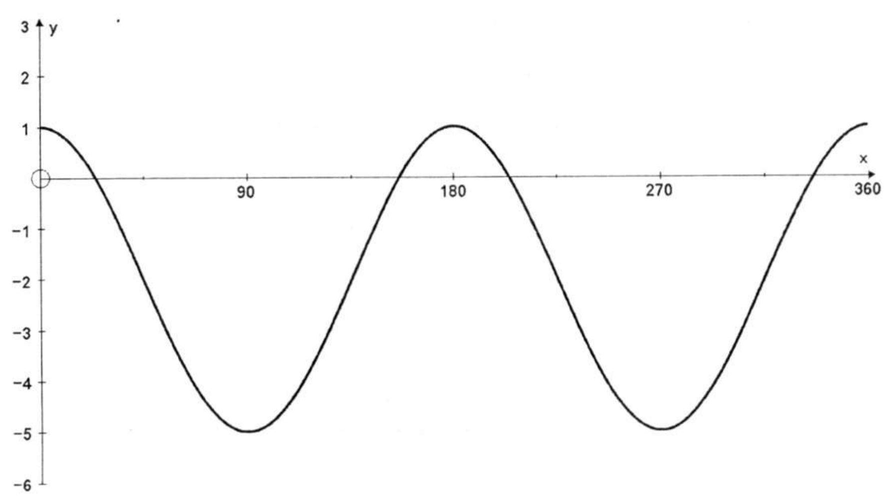
$t > \ln 10\left(\frac{7}{\ln 2}\right)$

$t > 23.253$ $(5s.f.)$ [$M1$]

The year in which its value first exceeded ten times the initial value is 1954. [A1]

8. (i) $a = 3$ [B1] $b = \frac{360°}{120°}$, $b = 3$ [B1] $c = 1 - 3$, $c = -2$ [B1]

(ii) 2 marks for graph. 1 mark for labels.



9. $x^2+3x-6=0$

$$\alpha+\beta=-3$$

$$\alpha\beta=-6 \qquad [M1]$$

$$x^2-6x+q=0$$

$$\frac{k}{\alpha^3}+\frac{k}{\beta^3}=6 \qquad [M1]$$

$$\frac{k(\alpha^3+\beta^3)}{\alpha^3\beta^3}=6$$

$$k=\frac{6\alpha^3\beta^3}{\alpha^3+\beta^3}$$

$$=\frac{6(\alpha\beta)^3}{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)} \qquad [M1]$$

$$=\frac{6(\alpha\beta)^3}{(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)}$$

$$=\frac{6(-6)^3}{(-3)((-3)^2-3(-6))}$$

$$=16 \qquad [A1]$$

$$\left(\frac{k}{\alpha^3}\right)\left(\frac{k}{\beta^3}\right)=q \qquad [M1]$$

$$q=\frac{k^2}{\alpha^3\beta^3}$$

$$q=\frac{16^2}{(-6)^3}$$

$$q=-\frac{32}{27} \qquad [A1]$$

10. (i) $f(1)=0$

$m-(5m-1)+(m+1)+m^2=0$ [M1]

$m^2-3m+2=0$

$(m-1)(m-2)=0$

$m=1 \quad or \quad m=2$ [M1]

$f(4)\neq 0$

$64m-16(5m-1)+4(m+1)+m^2\neq 0$ [M1]

$64m-80m+16+4m+4+m^2\neq 0$

$m^2-12m+20\neq 0$

$(m-2)(m-10)\neq 0$

$m\neq 2 \quad or \quad m\neq 10$ [A1]

$\therefore \quad m=1(shown)$

(ii) $x^3-4x^2+2x+1=(x-1)(x^2+bx-1)$ [M1]

Comparing coefficients of x,

$2=-1-b$

$b=-3$

$(x-1)(x^2-3x-1)=0$ [M1]

$x=1 \quad or \quad x=\dfrac{-(-3)\pm\sqrt{(-3)2-4(1)(-1)}}{2(1)}$

$=\dfrac{3\pm\sqrt{13}}{2}$

$x=1 \quad or \quad x=-0.30\ (2d.p.) \quad or \quad x=3.30\ \ (2d.p.)$ [A2]

11. (a) $y=(2k-1)x^2+2k+4 \quad ---(1)$

$y=3kx \quad ---(2)$

$(1)=(2),$

$(2k-1)x^2+2k+4=3kx$

$(2k-1)x^2-3kx+2k+4=0$ [M1]

For 1 real root,

Discriminant = 0

$(-3k)^2-4(2k-1)(2k+4)=0$ [M1]

$9k^2-4(4k^2+8k-2k-4)=0$

$9k^2-16k^2-24k+16=0$

$7k^2+24k-16=0$ [M1]

$(7k-4)(k+4)=0$

$k=\dfrac{4}{7} \quad or \quad k=-4$ [A1]

(b) $(h+3)x^2 - 3x > x + h$

$(h+3)x^2 - 4x - h > 0$

For $(h+3)x^2 - 4x - h$ to be always positive,

Discriminant < 0

$(-4)^2 - 4(h+3)(-h) < 0$ [M1]

$16 + 4h^2 + 12h < 0$

$h^2 + 3h + 4 < 0$ [M1]

$(h+1.5)^2 + 1.75 < 0$ [M1]

Since $(h+1.5)^2 + 1.75$ is always positive,

there are no values of h such that $(h+1.5)^2 + 1.75 < 0$. [A1]

(c) $2x^2 + p = 2(x-1)$

$2x^2 - 2x + p + 2 = 0$

$$\begin{aligned}\text{Discriminant} &= (-2)^2 - 4(2)(p+2)\\ &= 4 - 8p - 16\\ &= -12 - 8p\end{aligned}$$ [M1]

If $p > -\frac{3}{2}$, then

$2p > -3$

$-8p < 12$

$-12 - 8p < 0$ [M1]

Since discriminant < 0, $2x^2 + p = 2(x\text{-}1)$ has no real roots. [A1]

12. (i) $2x^3-31x-27=A(x-4)(x+2)^2+Bx+C$

Sub $x=4$,

$2(4)^3-31(4)-27=4B+C$

$4B+C=-23$ ---(1)

Sub $x=-2$,

$2(-2)^3-31(-2)-27=-2B+C$

$-2B+C=19$ ---(2) [M1]

(1) − (2) $6B=-42$

$B=-7$ [A1]

Sub $B=-7$ into (1),

$4(-7)+C=-23$

$C=5$ [A1]

Sub $x=0$, $B=-7$, $C=5$,

$-27=A(-4)(2)^2+5$

$A=2$ [A1]

$\therefore$ $A=2$, $B=-7$, $C=5$

(ii) $2x^3-31x-27=2(x-4)(x+2)^2-7x+5$

$$\frac{2x^3-31x-27}{(x-4)(x+2)^2}=2+\frac{-7x+5}{(x-4)(x+2)^2}$$

Let $\frac{-7x+5}{(x-4)(x+2)^2}=\frac{P}{x-4}+\frac{Q}{x+2}+\frac{R}{(x+2)^2}$ [M1]

$-7x+5=P(x+2)^2+Q(x-4)(x+2)+R(x-4)$

Sub $x=-2$,

$-7(-2)+5=-6R$

$R=-\frac{19}{6}$ [M1]

Sub $x=4$,

$-7(4)+5=36P$

$P=-\frac{23}{36}$ [M1]

Sub $x=0$,

$5=4\left(-\frac{23}{36}\right)-8Q-4\left(-\frac{19}{6}\right)$

$Q=\frac{23}{36}$ [M1]

$$\therefore \quad \frac{2x^3-31x-27}{(x-4)(x+2)^2}=2-\frac{23}{36(x-4)}+\frac{23}{36(x+2)}-\frac{19}{6(x+2)^2}$$ [A1]