CEDAR GIRLS' SECONDARY SCHOOL
End-of-Year Examination
Secondary Three

ADDITIONAL MATHEMATICS

4047

5 October 2016

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper and Graph Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 The length and the width of a rectangular fish tank are $\left(\sqrt{2}+1\right)$ m and $\left(5-\sqrt{8}\right)$ m respectively. Given that the volume of the tank is 5 m^3, express the height of the tank in the form $a\sqrt{2}+b$ m, where a and b are constants. [4]

2 On the same diagram, sketch the graphs of $y^2 = 4x$ and $y = x^{\frac{3}{2}}$ for $x \geq 0$.
Find the x-coordinates of the points of intersection of the 2 graphs. [5]

3 The roots of the quadratic equation $2x^2 - 4x + 7 = 0$ are α and β.

(i) Find the value of

(a) $(\alpha+1)(\beta+1)$, [2]

(b) $\alpha^3 + \beta^3$. [2]

(ii) Find the quadratic equation whose roots are $\frac{\alpha^2}{\beta+1}$ and $\frac{\beta^2}{\alpha+1}$. [3]

4 The polynomial f(x) has degree of 4 and the coefficient of x^4 is 2. The roots of f(x) = 0 are 1, – 2 and a repeated root k, where k is a positive constant. f(x) has a remainder of 32 when divided by $x - 2$.

(i) Find the value of k. [3]

(ii) Hence, find an expression for f(x) in descending powers of x. [2]

5 Express $\frac{4x^3 + 5x - 2}{x^4 + x^2}$ as the sum of 3 partial fractions. [6]

6 **(a)** Sketch the graph of $y = \frac{2}{e^{2x}}$. [2]

(b) Solve the equation $\log_3(3x^2 - 6) - \log_{\sqrt{3}}(x-1) = 1$. [4]

(c) Given that $y = 2^x$, express $2^{3x-1} - 4^{3x} + 8^{x+1}$ in terms of y. [3]

7 The equation of a curve is $y = x^2 + kx + 8 - k$, where k is a constant.

(i) Find the range of values of k for which $y > 0$ for all real values of x. [3]

(ii) Given that $k = -1$, find the values of m for which the line $y = mx - 5$ is a tangent to the curve. [3]

8 **(a)** Find all the angles between 0 and 2π inclusive which satisfy the equation $3 + 2\sin x = 3\cos^2 x$. [4]

(b) Solve, for $0° \leq x \leq 360°$, the equation $\sec(2x - 50°) = 1.5$. [3]

9 **(a)** Find the exact value of $\cos 120° + \operatorname{cosec} 315°$. [3]

(b) Prove the identity $(\sec x - \tan x)(\sin x + 1) = \cos x$. [3]

(c) Given that $\sin x = \frac{5}{13}$ and $\tan y = -\frac{3}{4}$ and that x and y lie in the same quadrant, find the exact value of $\tan x + \cos y$. [4]

10 The function f is defined by $f(x) = 2\cos\frac{3}{2}x + 1$ for $0° \leq x \leq 360°$.

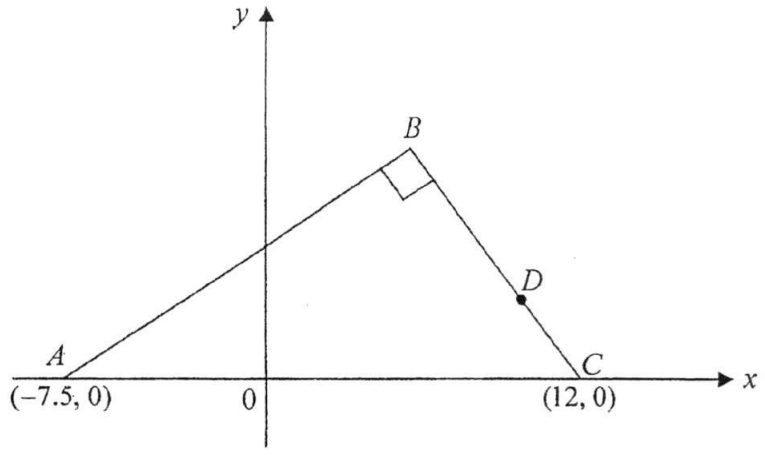
(i) State the period and amplitude of f. [2]

(ii) Find the coordinates of the maximum and minimum points of the function f. [3]

(iii) Find the x-coordinates of the points where the curve meets the x axis. [3]

(iv) Hence, sketch the graph of $y = f(x)$ for $0° \leq x \leq 360°$. [3]

11 Solutions to this question by accurate drawing will not be accepted.



The diagram above shows a right-angled triangle ABC. A is the point $(-7.5, 0)$, C is the point $(12, 0)$ and $\angle ABC$ is a right angle. The line AB is parallel to the straight line $2x - 3y = 10$.

(i) Find the coordinates of B. [5]

D is a point on the line BC such that the ratio of BD to DC is 2 : 1.

(ii) State the ratio of the area of triangle ABD to the area of triangle ABC. [1]

(iii) Find the coordinates of D. [2]

(iv) Calculate the area of triangle ABD. [2]

12 A circle C_1 passes through the points $X(1, 3)$ and $Y(5, 5)$, where XY is a diameter of the circle.

(i) Find the equation of the circle C_1. [3]

A second circle C_2 is a reflection of circle C_1 with $y = 2$ as the line of reflection.

(ii) Find the equation of the circle C_2. [2]

A third circle C_3 has the equation $x^2 + y^2 - 6x + 8y - 5 = 0$.

(iii) Find the coordinates of the centre and the radius of the circle C_3. [2]

(iv) Showing your working clearly, explain whether circle C_3 will intersect with circle C_1. [2]

13 Answer the whole of this question on a sheet of graph paper.

The variables x and y are related by the equation $y = \frac{a^x}{e^{3b}}$, where a and b are constants. The table below shows values of x and y.

x	1.0	1.5	2.0	2.5	3.0
y	0.05	0.22	1.00	4.48	20.09

(i) Draw a straight line graph of ln y against x, using a scale of 2 cm to represent 0.5 unit on the horizontal axis and 2 cm to represent 1 unit on the vertical axis. [2]

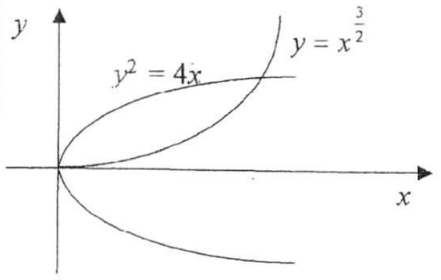
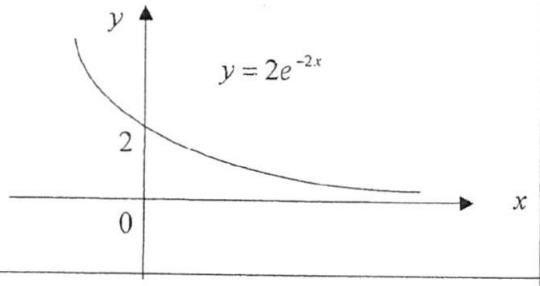
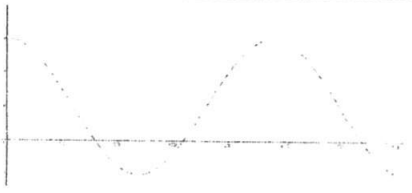
(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Use your graph to estimate the value of x when $y = 6$. [2]

(iv) By drawing a suitable straight line on the same diagram, find the value of x for which $(ea)^x = e^{3b-1}$. [3]

End Of Paper

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 3 ADDITIONAL MATHEMATICS 4047
2016 End-Of-Year Examination
Answer Key

1	$\frac{15}{17}\sqrt{2}-\frac{5}{17}$
2	 $x=0, x=2, x=-2$ *(rejected)*
3ia	6.5
3ib	-13
3ii	$26x^2+64x+49=0$
4i	$k=4$
4ii	$2x^4-14x^3+12x^2+64x-64$
5	$\frac{4x^3+5x-2}{x^4+x^2}=\frac{5}{x}-\frac{2}{x^2}+\frac{2-x}{x^2+1}$
6a	
6b	$x=1.5$
6c	$8\frac{1}{2}y^3-y^6$
7i	$-8<k<4$
7ii	$m=6.48, \quad m=-8.48 \quad (3s.f.)$
8a	$x=0, \pi, 3.87, 5.55, 2\pi$
8b	$x=0.9°, 49.1°, 180.9°, 229.1° \quad (1\,d.p.)$
9a	$-\frac{1}{2}-\sqrt{2}$
9b	Proving
9c	$-1\frac{13}{60}$
10i	Amplitude =2 Period = 240°
10ii	Maximum points= (0°, 3), (240°, 3) Minimum points= (120°, – 1), (360°, – 1)
10iii	$x=80°, 160°, 320°$
10iv	
11i	$B=(6, 9)$
11ii	2 : 3
11iii	$D=(10, 3)$
11iv	58.5 units^2
12i	$(x-3)^2+(y-4)^2=5$
12ii	$(x-3)^2+y^2=5$
12iii	Centre = $(3, -4)$ Radius = $\sqrt{30}$ units
12iv	$r_1+r_3=7.71<8$ Hence C_3 will not intersect C_1.
13ii	$a=20.1$ (17.1 to 23.4) $b=-2$ (-1.96 to 2.04)
13iii	$x=2.6$ (2.55 to 2.65)
13iv	$x=1.25$ (1.2 to 1.3)