CHIJ ST. THERESA'S CONVENT
END-OF-YEAR EXAMINATION 2016
SECONDARY 3 EXPRESS

ADDITIONAL MATHEMATICS

4047

05 Oct 2016

2 hours 30 minutes

Additional Material: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your index number and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

<u>Hand in questions 1 to 7 separately from questions 8 to 13</u>

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

$R = \sqrt{a^2 + b^2}$

$\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

1 Solve

(a) $4^{y-2} = 4\left(9^{3-y}\right)$. [3]

(b) $|4x - 7| = 6x$. [3]

2 (a) The curve $y = a(x+3)^b$ passes through the points $(0,\ 4)$, $(1,\ 3)$ and $(-6,\ k)$. Find the exact values of a, b and k. [4]

(b) Solve the equation $16^x = 66 - 4^{x-1}$. [4]

3 In the expansion of $\left(\frac{x}{2} + \frac{k}{x^2}\right)^9$ where k is a positive constant, the term independent of x is $10\frac{1}{2}$.

(i) Show that $k = 2$. [4]

(ii) With this value of k, find the coefficient of x^6 in the expansion of $\left(2x^6 - 128\right)\left(\frac{x}{2} + \frac{k}{x^2}\right)^9$. [4]

4 (i) Sketch the graph of $y = |2x - 5| - 3$, indicating clearly the coordinates of the turning point and of the points where the graph meets the x and y-axes. [4]

(ii) Hence, find the range of values of x for which $y < 0$. [1]

5 (a) Find the values of k for which the line $kx - 2y = 0$ is a tangent to the curve $y = x^2 - 3x + 4$. [4]

(b) A curve has the equation $y = kx^2 - 14x + 4k + 21$, where k is a constant. Find the range of values k for which $y > 0$ for all values of x. [4]

6 (i) Express $\dfrac{11-x}{(x-3)(x+5)}$ in partial fractions. [4]

(ii) Hence, solve the equation $\dfrac{11-x}{(x-3)(x+5)} + \dfrac{2}{x+5} = 4$. [2]

7 (a) Prove the identity $\dfrac{\sin 2\theta + \cos\theta}{\cos 2\theta - \sin\theta - 1} \equiv -\cot\theta$. [4]

(b) Given that $\sin x = -\dfrac{3}{5}$ and $\tan x > 0$, find, without using calculators, the value of

(i) $\cos x$, [1]

(ii) $\sin 2x$, [2]

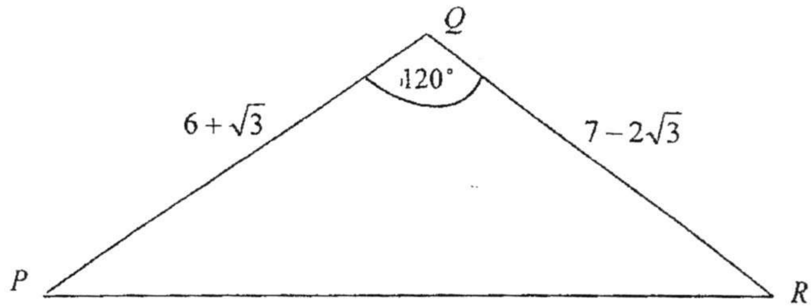
(iii) $\cos \dfrac{1}{2}x$. [2]

Start Question 8 on a fresh sheet of Answer Paper.

Hand in Questions 8 to Question 13 separately from Question 1 to Question 7.

8 In triangle PQR, $PQ = 6+\sqrt{3}$ cm, $QR = 7-2\sqrt{3}$ cm and $\angle PQR = 120^\circ$.

Express the area of PQR in the form ($p+q\sqrt{3}$), where p and q are rational numbers. . [4]

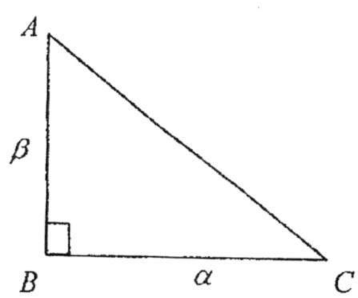


9 The roots of the quadratic equation $2x^2-6x+1=0$ are α and β.

Find $\alpha^2+\beta^2$ and hence, find [3]

(i) the quadratic equation whose roots are α^3 and β^3, [2]

(ii) the perimeter of a right-angled triangle ABC in the form $a+b\sqrt{2}$, if α and β represent the lengths, in cm, of the two shorter sides of the triangle as shown in the diagram below. [2]



10 The expression $f(x) = 3x^3 + ax^2 + bx - 3$, where a and b are constants, has a factor $x - 1$ and leaves a remainder of 33 when divided by $x - 4$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and b found in part **(i)**, show that $f(x)$ may be expressed in the form of $f(x) = (x-1)(3x^2 + px + q)$, where p and q are constants. [2]

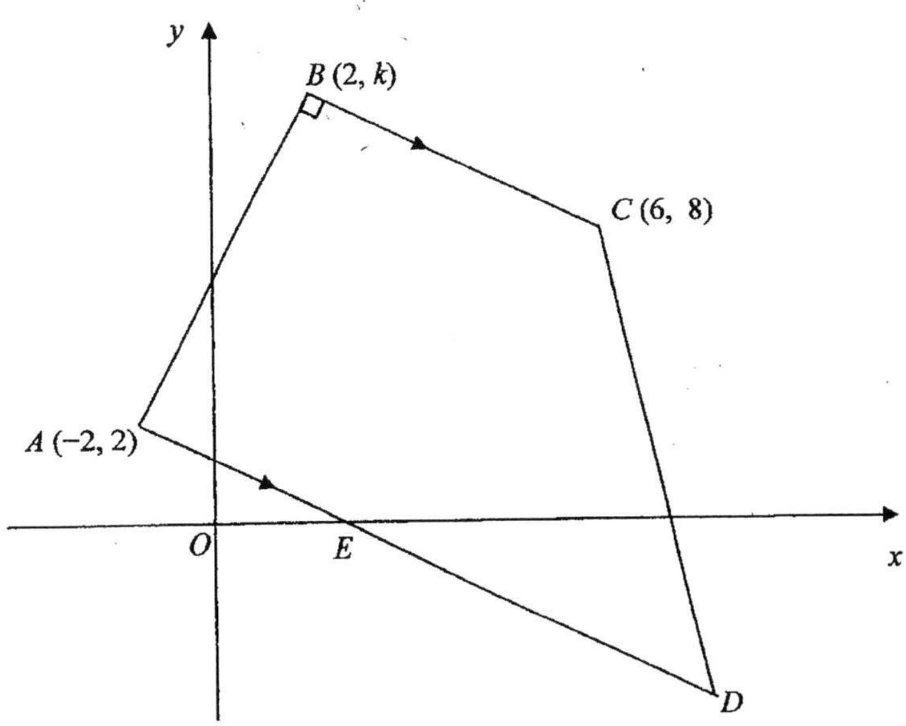
(iii) Solve $f(x) = 0$ and hence, solve the equation $\frac{3}{8}x^3 - \frac{13}{4}x^2 + \frac{13}{2}x - 3 = 0$ [5]

11 **Solutions to this question by accurate drawing will not be accepted.**

The diagram (not drawn to scale) shows a trapezium $ABCD$ in which AD is parallel to BC and AB is perpendicular to BC.

The coordinates of A, B and C are $(-2, 2)$, $(2, k)$ and $(6, 8)$ respectively.

AD cuts the x-axis at E and the gradient of CD is -3.



(i) Given that k is positive, find the value of k. [3]

(ii) Find the coordinates of E. [2]

(iii) Find the coordinates of D and hence, find the area of the trapezium $ABCD$. [4]

12 **(a)** Solve, for $0^\circ \le x \le 360^\circ$, the equation $2\sec^2 x = 5\tan x$. [4]

(b) Given that $y = 5\cos\theta + 2\sin\theta$, express $5\cos\theta + 2\sin\theta$ in the form of $R\cos(\theta - \alpha)$. [2]

Hence, for $0^\circ \le \theta \le 360^\circ$,

(i) state the maximum value of y and the corresponding value of θ. [2]

(ii) find the value of the acute angle θ when $y = 4$. [2]

13 **(i)** A curve has the equation $y = \cos 2x + 1$.

State the

(a) amplitude of the curve. [1]

(b) period of the curve. [1]

(c) maximum and minimum values of the curve. [1]

(ii) Sketch on the same diagram, the graphs of $y = \cos 2x + 1$ and $y = -2\sin x$ for the interval $0^0 \le x \le 360^0$. [4]

Hence,

(a) state the number of solutions for $\cos 2x + 1 = -2\sin x$ for $0^0 \le x \le 360^0$. [1]

(b) find the value of k given the equation $\cos 2x + k = -2\sin x$ has only one solution of x for $0^0 \le x \le 360^0$. [1]

End of Paper

(Have you checked your work?)

Sec 3 Exp. Add. Maths EOY 2016

1a) $4^{y-2} = 4(9^{3-y})$

$\frac{4^y}{16} = 4\left(\frac{9^3}{9^y}\right)$ M1

$36^y = 4(9)^3(16)$

$= 4(9)^3(4^2)$

$= (4^3)(9^3)$ M1

$= 36^3$

$\therefore y = 3$ A1

b) $|4x-7| = 6x$ M1

$4x-7 = 6x$ or $4x-7 = -6x$

$-7 = 2x$ $\quad$ $10x = 7$

$x = -3\frac{1}{2}$ (NA) A1 $\quad$ $x = \frac{7}{10}$ A1

6

2a) $y = a(x+3)^{b}$ $(0,4)$ $(1, 3)$ $(-6, k)$

$$4 = a(3)^{b} \quad — ①$$
$$3 = a(4)^{b} \quad — ②$$
$$k = a(-3)^{b} \quad — ③$$

1

$\frac{②}{①}$ $\frac{3}{4} = \left(\frac{4}{3}\right)^{b}$

$\left(\frac{4}{3}\right)^{b} = \left(\frac{4}{3}\right)^{-1}$

$\therefore \quad b = -1$ 1

sub → ① $4 = a(3)^{-1}$

$12 = a$ 1

sub → ③ $k = 12(-3)^{-1}$

$= \frac{12}{-3}$

$= -4$ 1

4

Ans: $a = 12, \quad b = -1, \quad k = -4$

b) $16^{x} = 66 - 4^{x-1}$

$(4^{x})^{2} = 66 - \frac{4^{x}}{4}$ 1

Let $y = 4^{x}$

$\therefore \quad y^{2} = 66 - \frac{y}{4}$ 1

$4y^{2} + y - 264 = 0$

$(y-8)(4y+33) = 0$

$y = 8$ or $y = -\frac{33}{4}$ (NA) 1

$2^{2x} = 2^{3} \Rightarrow \underline{x = \frac{3}{2}}$ 1

4

8

3) $\left(\frac{x}{2} + \frac{k}{x^2}\right)^9$

$$G T = {}^9C_r \left(\frac{x}{2}\right)^{9-r} \left(\frac{k}{x^2}\right)^r$$

$$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} k^r x^{9-r} x^{-2r}$$

$$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} k^r x^{9-3r} \qquad 1$$

When $9 - 3r = 0$

$$\Rightarrow r = 3 \qquad 1$$

$$\therefore {}^9C_3 \left(\frac{1}{2}\right)^{9-3} k^3 = 10\tfrac{1}{2}$$

$$84\left(\frac{1}{64}\right) k^3 = \frac{21}{2}$$

$$k^3 = \frac{21}{2}(64)\left(\frac{1}{84}\right) \qquad 1$$

$$= 8$$

$$\underline{\underline{k = 2}} \qquad 1 \quad 4$$

(ii) $(2x^6 - 128)\left(\frac{x}{2} + \frac{2}{x^2}\right)^9$

When $9 - 3r = 6 \Rightarrow 3r = 3 \Rightarrow r = 1 \qquad 1$

Term in x^6 is ${}^9C_1 \left(\frac{1}{2}\right)^8 2^1 x^6$

$$\frac{18}{256} x^6 \qquad 1$$

$$\therefore (2x^6 - 128)\left(\frac{x}{2} + \frac{2}{x^2}\right)^9$$

$$= (2x^6 - 128)\left(\ldots\ldots + 10\tfrac{1}{2} + \frac{18}{256}x^6 + \ldots\right)$$

$$= \ldots\ldots 2\left(\frac{21}{2}\right)x^6 - 128\left(\frac{18}{256}x^6\right) + \ldots\ldots$$

$$= 21x^6 - 9x^6 \qquad 1$$

$$= 12x^6 \qquad \therefore \text{Coeff of } x^6 \underline{\underline{= 12}} \qquad 1 \quad 4$$

8

1

4 i) $y = |2x-5| - 3$

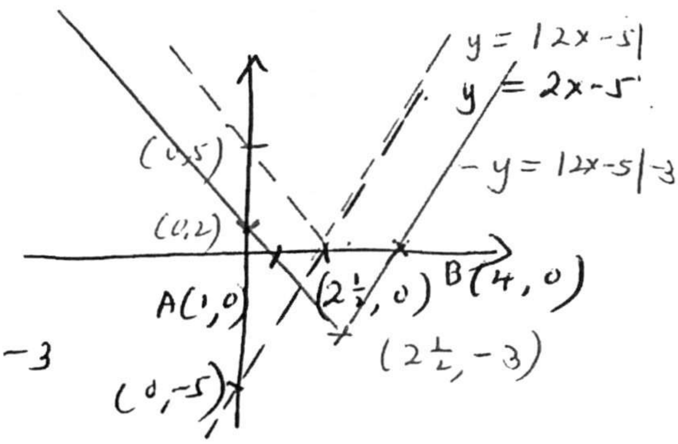
when $|2x-5| - 3 = 0$

$|2x-5| = 3$

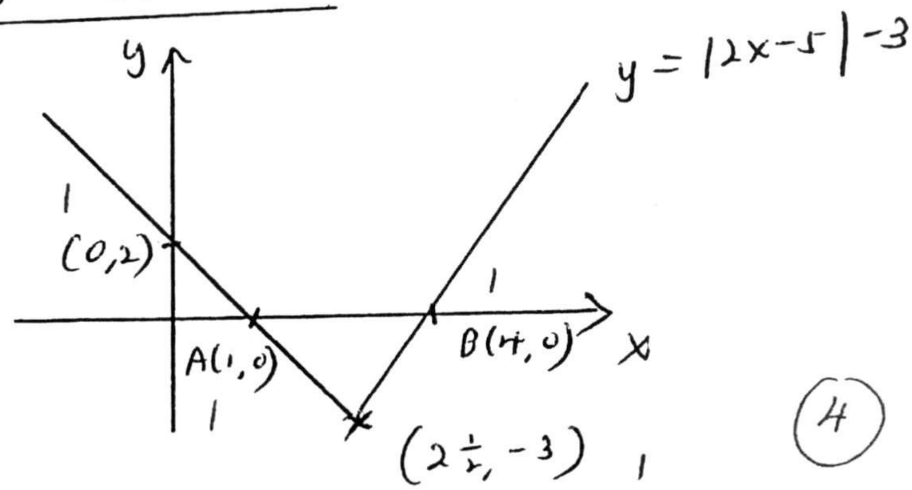
$2x-5 = 3$ or $2x-5 = -3$

$2x = 8$ $\quad$ $2x = 2$

$x = 4$ $\quad$ $x = 1$



$\therefore$ Final sketch



(4)

(ii) For $y < 0$, $\underline{\underline{1 < x < 4}}$ (1)

5 a) $kx - 2y = 0$ — ①

$y = x^2 - 3x + 4$ — ②

Sub ② → ①

$kx - 2(x^2 - 3x + 4) = 0$

$kx - 2x^2 + 6x - 8 = 0$

$2x^2 - kx - 6x + 8 = 0$

$2x^2 - (k+6)x + 8 = 0$ 1

Since $b^2 - 4ac = 0$

$[-(k+6)]^2 - 4(2)(8) = 0$ 1

$(k+6)^2 = 64$

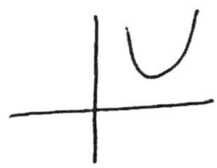
1

$k + 6 = \pm 8$ 4

$\underline{k = 2 \text{ or } -14}$ 1

b) $y = kx^2 - 14x + 4k + 21$

Graph is U shape ⇒ $k > 0$ — ① 1



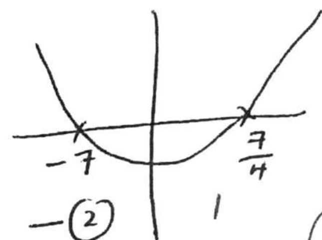
$b^2 - 4ac < 0$ 1

$\therefore (-14)^2 - 4(k)(4k+21) < 0$

$196 - 4k(4k+21) < 0$

$196 - 16k^2 - 84k < 0$

$16k^2 + 84k - 196 > 0$



$4k^2 + 21k - 49 > 0$

$(4k - 7)(k + 7) > 0$

8

$k < -7$ or $k > \frac{7}{4}$ — ② 1 4

From ① & ② $\underline{k > \frac{7}{4}}$ Ans. 1

S

6 i.) $$\frac{11-x}{(x-3)(x+5)} = \frac{A}{x-3} + \frac{B}{x+5}$$ 1

$$= \frac{A(x+5) + B(x-3)}{(x-3)(x+5)}$$

$$\therefore \; 11-x = A(x+5) + B(x-3)$$ 1

when $x = -5$: $16 = -8B$

$$\Rightarrow B = -2$$ 1

when $x = 3$ $8 = 8A$

$$\Rightarrow A = 1$$ 1

$$\therefore \; \frac{11-x}{(x-3)(x+5)} = \underline{\underline{\frac{1}{x-3} - \frac{2}{x+5}}}$$ 4

ii.) $$\frac{11-x}{(x-3)(x+5)} + \frac{2}{x+5} = 4$$

$$\therefore \; \frac{1}{x-3} - \frac{2}{x+5} + \frac{2}{x+5} = 4$$

$$\therefore \; \frac{1}{x-3} = 4$$ 1

$$\frac{1}{4} = x-3$$

$$\underline{\underline{x = 3\tfrac{1}{4}}}$$ 1 2

6

7) a) To prove $\dfrac{\sin 2\theta + \cos\theta}{\cos 2\theta - \sin\theta - 1} = -\cot\theta$

Proof

$$LHS = \frac{2\sin\theta\cos\theta + \cos\theta}{1 - 2\sin^2\theta - \sin\theta - 1} \qquad B1,\ B1$$

$$= \frac{\cos\theta\,(2\sin\theta + 1)}{-2\sin^2\theta - \sin\theta}$$

$$= \frac{\cos\theta\,(2\sin\theta + 1)}{-\sin\theta\,(2\sin\theta + 1)} \qquad 1$$

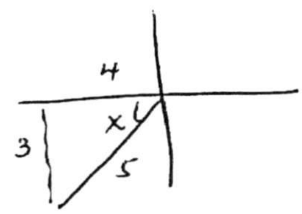
$$= -\frac{\cos\theta}{\sin\theta} \qquad 1$$

$$= -\cot\theta \qquad \text{proved.} \quad 4$$

b) $\sin x = -\frac{3}{5}$

$\cos x = -\frac{4}{5}$

$\tan x = \frac{3}{4}$



(i) $\cos x = -\frac{4}{5}$ 1

(ii) $\sin 2x = 2\sin x\cos x$

$= 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)$ 1

$= \frac{24}{25}$ 1

$\cos x = 2\cos^2\frac{1}{2}x - 1$

$\dfrac{\cos x + 1}{2} = \cos^2\frac{x}{2}$

$\dfrac{-\frac{4}{5} + 1}{2} = \cos^2\frac{x}{2}$

$\cos^2\frac{x}{2} = \frac{1}{10}$

$\cos\frac{x}{2} = \pm\frac{1}{\sqrt{10}}$ 1

Since $180^\circ < x < 270^\circ$

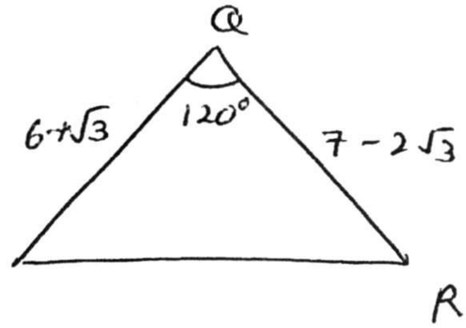
$90^\circ < \frac{x}{2} < 135^\circ$

$\Rightarrow \frac{x}{2}$ in 2^{nd} Quad.

5 $\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{10}}$ 1

9

8)



$$\text{Area} = \text{`}\tfrac{1}{2}ab\sin C\text{'}$$

$$= \tfrac{1}{2}(6+\sqrt{3})(7-2\sqrt{3})\sin 120^\circ \qquad 1$$

$$= \tfrac{1}{2}(6+\sqrt{3})(7-2\sqrt{3})\frac{\sqrt{3}}{2}$$

$$= \tfrac{1}{4}(6\sqrt{3}+3)(7-2\sqrt{3}) \qquad 1$$

$$= \tfrac{1}{4}(42\sqrt{3} - 36 + 21 - 6\sqrt{3})$$

$$= \tfrac{1}{4}(36\sqrt{3} - 15) \qquad 1$$

$$= 9\sqrt{3} - \frac{15}{4} \qquad 1$$

4

9) $2x^2 - 6x + 1 = 0$

$\alpha + \beta = 3$ 1

$\alpha\beta = \frac{1}{2}$ 1

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 9 - 1$

$= 8$ 1

(i) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (3)(8 - \frac{1}{2})$

$= 3(\frac{15}{2})$

$= \frac{45}{2}$ 1

$\alpha^3\beta^3 = (\frac{1}{2})^3$

$= \frac{1}{8}$

The eq^n is $x^2 - \frac{45}{2}x + \frac{1}{8} = 0$ 1

(ii) $AC = \sqrt{\alpha^2 + \beta^2}$

$= \sqrt{8}$

$= 2\sqrt{2}$ 1

Perimeter $= \alpha + \beta + AC$

$= 3 + 2\sqrt{2}$ cm 1

7

10 i) $f(x) = 3x^3 + ax^2 + bx - 3$

$f(1) = 0$ — (1)

$f(4) = 33$ — (2)

From (1) $3 + a + b - 3 = 0 \Rightarrow a + b = 0$ — (3) 1

From (2) $192 + 16a + 4b - 3 = 33$

$16a + 4b = -156$

$4a + b = -39$ — (4) 1

(4) − (3) $3a = -39$

$\underline{\underline{a = -13}}$ 1

$\therefore \underline{b = 13}$ 1

(4)

(ii) $f(x) = 3x^3 - 13x^2 + 13x - 3$

	3	−13	13	−3	
+)	0	3	−10	+3	1
	3	−10	3	$\boxed{0}$	

1

(2)

$\therefore f(x) = \underline{(x-1)(3x^2 - 10x + 3)}$ 1

(iii) $f(x) = 0 \Rightarrow \underline{\underline{x = 1}}$ or $3x^2 - 10x + 3 = 0$

$(3x-1)(x-3) = 0$ 1

$\underline{\underline{x = \frac{1}{3} \text{ or } x = 3}}$ 1

$\frac{3}{8}x^3 - \frac{13}{4}x^2 + \frac{13}{2}x - 3 = 0$

$3\left(\frac{1}{2}x\right)^3 - 13\left(\frac{1}{2}x\right)^2 + 13\left(\frac{1}{2}x\right) - 3 = 0$ 1

Let $y = \frac{x}{2}$

$\therefore 3y^3 - 13y^2 + 13y - 3 = 0$

$\therefore y = 1$ or $y = \frac{1}{3}$ or $y = 3$ 1

$\frac{x}{2} = 1$ $\frac{x}{2} = \frac{1}{3}$ $\frac{x}{2} = 3$

$\underline{\underline{x = 2 \quad x = \frac{2}{3} \quad x = 6}}$ 1

(11) (5)

11) i) Grad of AB $= \dfrac{k-2}{2+2}$

$= \dfrac{k-2}{4}$

Grad of BC $= \dfrac{k-8}{2-6}$

$= \dfrac{k-8}{-4}$ 1

$\therefore \left(\dfrac{k-2}{4}\right)\left(\dfrac{k-8}{-4}\right) = -1$ 1

$(k-2)(k-8) = 16$

$k^2 - 10k + 16 - 16 = 0$

$k(k-10) = 0$

$k = 0$ or $\underline{k = 10}$ 1 (3)

NA

(ii) Grad of BC $= \dfrac{10-8}{-4}$

$= -\dfrac{1}{2}$

Eqn of AD is $\dfrac{y-2}{x+2} = -\dfrac{1}{2}$

$2y - 4 = -x - 2$ 1

$2y = -x + 2$ — (1)

when $y = 0$, $x = 2$ $\Rightarrow \underline{E \equiv (2, 0)}$ 1

(2)

(iii) Eqn of CD is $\dfrac{y-8}{x-6} = -3$

$y - 8 = -3x + 18$

$y = -3x + 26$ — (2) 1

Sub (2) → (1)

$-6x + 52 = -x + 2$

$50 = 5x$

$x = 10$ 1

$\therefore y = -4$

$\therefore \underline{D \equiv (10, -4)}$ 1

(9)

Area of ABCD $= \dfrac{1}{2}\begin{vmatrix} -2 & 10 & 6 & 2 & -2 \\ 2 & -4 & 8 & 10 & 2 \end{vmatrix}$

$= \dfrac{1}{2}\left| 152 + 8 \right|$ 1

$= \underline{80 \text{ unit}^2}$ 1

(4)

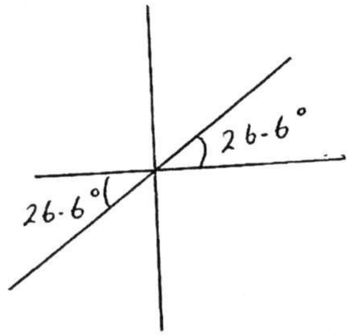
12a) $2\sec^2 x = 5\tan x$

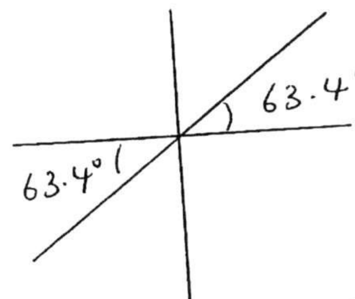
$2(1+\tan^2 x) = 5\tan x$

$2\tan^2 x - 5\tan x + 2 = 0$.

$(2\tan x - 1)(\tan x - 2) = 0$

$\tan x = \frac{1}{2}$ or $\tan x = 2$.





$x = 26.6°, 206.6°$

$x = 63.4°, 243.4°$

b) $y = 5\cos\theta + 2\sin\theta$.

$= R\cos(\theta - \alpha)$

$\tan\alpha = \frac{2}{5}$

$R = \sqrt{5^2 + 2^2}$

$\alpha = 21.8°$

$= \sqrt{29}$

$y = \sqrt{29}\cos(\theta - 21.8°)$

bi) $\max y = \sqrt{29}$ when $\theta - 21.8014° = 0$.

$\theta = 21.8°$

bii) $\sqrt{29}\cos(\theta - 21.8014°) = 4$

$\cos(\theta - 21.8014°) = \frac{4}{\sqrt{29}}$

$\theta - 21.8014° = 42.0311°$

$\theta \approx 63.8°$.

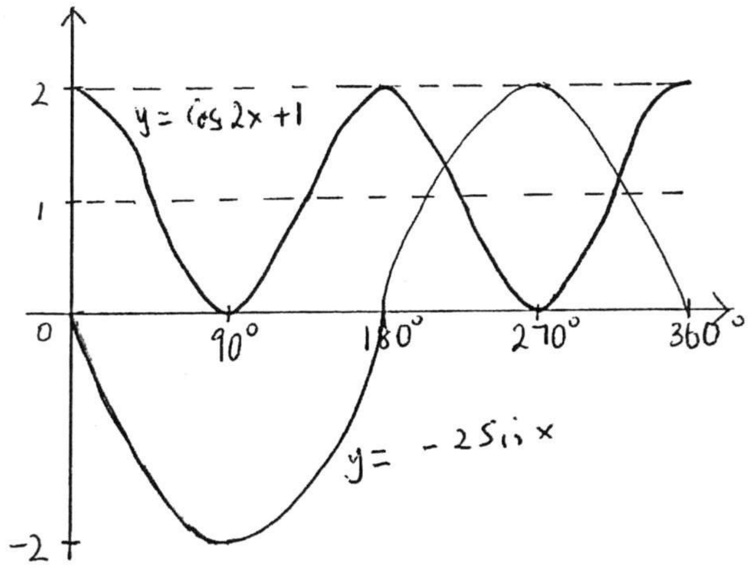
13i) $y = \cos 2x + 1$

a) Amplitude $= 1$

b) Period $= 180°$

c) Max $= 2$ min $= 0$

ii)



a) $\cos 2x + 1 = -2\sin x$

There are $\underline{\underline{2}}$ solutions.

b) $\cos 2x + K = -2\sin x$

$K = 3$.