Name : ________________________________ Register Number : ______

Class : ______

Clementi Town Secondary School
End-of-Year Examination 2016
Secondary 3 Express

ADDITIONAL MATHEMATICS

4047
5 October 2016
2 hours 30 minutes

Additional Materials provided: Answer Paper (7 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open the booklets until you are told to do so.
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the answer paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or in 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

Answer ***all*** questions on the answer paper provided.

1 Express $\dfrac{4x+7}{x^2+6x+9}$ in partial fractions. [4]

2 Find the value of m and of n such that $\dfrac{\sqrt{5}-\sqrt{3}}{2\sqrt{5}+5\sqrt{3}} = m+n\sqrt{15}$. [4]

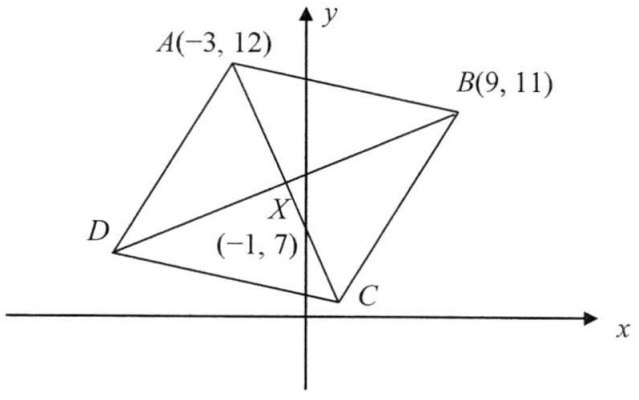
3 Find the first three terms in the expansion of $\left(3-x^2\right)^4$ in ascending powers of x.
Hence find the coefficient of x^4 in the expansion of $\left(1+x^2\right)\left(3-x^2\right)^4$. [5]

4 **(i)** Sketch the graph of $y = 1+|3-2x|$ for $-1 \le x \le 3$. [3]

(ii) State the range of values of x for which $y < 2$. [2]

(iii) Find the range of values of m for which $1+|3-2x| = m$ has two real roots. [1]

5 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram above, $ABCD$ is a rhombus. A and B are $(-3, 12)$ and $(9, 11)$ respectively.
The diagonals of the rhombus intersect at $X(-1, 7)$.
Find

(i) the equation of line AC, [2]

(ii) coordinates of D, [2]

(iii) area of rhombus $ABCD$. [3]

[Turn over

6 The function f is defined, for $0 \leq x \leq 2\pi$, by $f(x) = 1 + 3\sin x$.

(i) State the amplitude and the period of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

(iii) State the coordinates of the maximum point of the curve $y = f(x)$. [1]

(iv) Find the range of x when $y > 1$. [2]

7 **(a)** Given that $4x^3 - 6x^2 + ax + 3$ leaves a remainder of 7 when divided by $2x - 1$, find the value of a. [3]

(b) Given that $3x^2 - 11x + 3 = A(x-2)(x-1) + B(x-1) + C$ for all values of x, find the values of A, B and C. [5]

8 The quadratic equation $2x^2 - 3x + 4 = 0$ has roots α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Find the quadratic equation whose roots are α^3 and β^3. [5]

9 **(a)** The straight line $y = 2p + 1$ intersects the curve $y = x + \frac{p^2}{x}$ at two distinct points. Find the range of values of p. [4]

(b) Find the range of values of k for which the straight line $y = 2x + k$ does not cut the curve $x^2 + y^2 = 20$. [5]

10 **(a)** Given that $\tan\theta = \frac{1}{p}$, where $180^\circ < \theta < 270^\circ$, express in terms of p,

(i) $\sin\theta$, [2]

(ii) $\cos(-\theta)$. [1]

(b) Solve, for angles between 0° and 360°, the equation $8\sin^2 x = 7$. [4]

(c) Solve, for angles between 0 and π, the equation $\tan(y - 0.2) = 1.2$. [2]

11 A circle, C, has equation $x^2 + y^2 - 10x + 6y + 9 = 0$.

(i) Find the coordinates of the centre of C and the radius of C. [2]

(ii) Give a reason why the y-axis is a tangent to C. [1]

(iii) The circle C crosses the x-axis at the point $P(1\ ,\ 0)$.
Show that the equation of the tangent to the circle C at P is $3y - 4x = -4$. [3]

(iv) Find the coordinates of the point where the circle C crosses the x-axis again. [2]

(v) Show that the point $S(6,\ 1)$ is inside the circle. [2]

12 **(a)** Solve $5^x = 6$. [2]

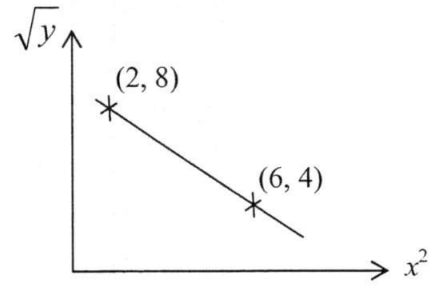
(b) Solve $e^x(2e^x - 1) = 10$. [4]

(c) Solve the simultaneous equations

$$\frac{27^x}{\sqrt{9^y}} = 3,$$

$$\log_2 x - 2 = \log_2 y .$$ [5]

13 (a)



The figure shows part of a straight line obtained by plotting $\sqrt{y}$ against x^2. The line passes through the points (2, 8) and (6, 4). Find y in terms of x. [3]

(b) Answer this part of the question on a single sheet of graph paper.

The table shows some experimental values of two variables, x and y, which are known to be related by the equation

$$y = \frac{a}{x} + \frac{b}{x^2}.$$

x	1.0	1.5	2.0	2.5	3.0
y	11.9	9.8	8.0	6.7	5.8

(i) Draw a straight line graph of xy against $\frac{1}{x}$, using a scale of 2 cm to represent 0.2 units on the $\frac{1}{x}$ - axis and 2 cm to represent 2 units on the xy - axis. [3]

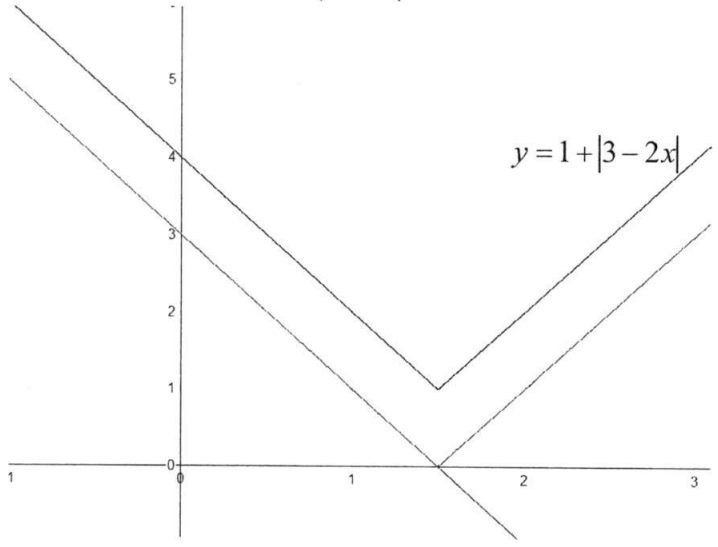
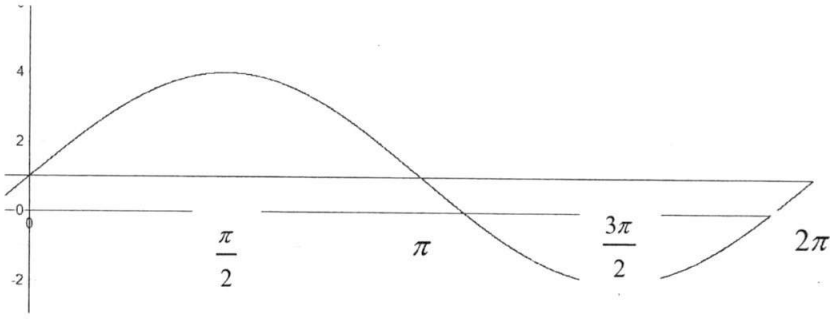
(ii) Use your graph to estimate

(a) the value of a and of b, [3]

(b) the value of x for which $y = \frac{13}{x}$. [2]

End of Paper

ANSWER SCHEME AM 3E 2016

1	$\frac{4}{x+3}-\frac{5}{(x+3)^2}$
2	$m=-\frac{5}{11}\quad n=\frac{7}{55}$
3	First three terms of $\left(3-x^2\right)^4$ $=81-108x^2+54x^4$ Coeff $=-54$
4i	Sketch the graph of $y=1+\lvert 3-2x\rvert$ for $-1\le x\le 3$. 
4ii	when $y<2$, Draw $y=2$ $1<x<2$
4iii	ANS $m>1$ for real roots
5i	$2y+5x=9$
5ii	$D=(-11,3)$
5iii	area of rhombus $ABCD$=116 sq units
6	 (iii) max pt is $(\frac{\pi}{2},4)$ (iii) when $y>1$ $0<x<\pi$

	(draw $y = 1$)
7(a)	$a = 10$
(b)	$A = 3$
8(i)	$= -1\frac{3}{4}$
8(ii)	Required eqn is $x^2 + \frac{45}{8}x + 8 = 0$ OR $8x^2 + 45x + 64 = 0$
9a	$p > -\frac{1}{4}$
9b	$k > 10$ OR $k < -10$
10a	
(i)	$\sin\theta = -\frac{1}{\sqrt{1+p^2}}$
(ii)	$\cos(-\theta) = -\frac{p}{\sqrt{1+p^2}}$
10b	$x = 69.3°,\ 110.7°$ OR $x = 249.3°,\ 290.7°$
10c	1.08 radians
11i	$r = 5$ units
11ii	y-axis is the tangent.
11iii	$3y - 4x = -4$
11iv	(9, 0)
11v	S is in the circle.
12a	$x = 1.11$(3s.f.)
12b	$x = 0.916$(3s.f.)
12c	$y = \frac{1}{11}$ $x = 4(\frac{1}{11}) = \frac{4}{11}$
13a	$y = (10 - x^2)^2$
13b	Answers on graph paper
	END OF PAPER