Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL

SEMESTRAL EXAMINATION TWO

SECONDARY THREE

ADDITIONAL MATHEMATICS **14 October 2016**

2 hours

Additional Materials:
Writing paper (6 sheets)

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This document consists of 5 printed pages and 1 blank page.

1. Without using the calculator, show that $\cos\left(\frac{5\pi}{12}\right)+\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}}{2}$. [4]

2. Express $\log_7 x-\log_{49}(x-2)=\log_3 1$ in the form $ax^2+bx+c=0$ and hence explain why there are no real solutions to the equation. [4]

3 The length and the width of a closed rectangular tank are $\left(1+\sqrt{2}\right)$m and $\left(3-\sqrt{8}\right)$m respectively. If the volume of the rectangular tank is 2 m^3, find the height of the tank in the form $\left(a+b\sqrt{2}\right)m$, where a and b are integers. [4]

4. If $\frac{1}{p}=\frac{\operatorname{cosec}x-1}{\cot x}$, prove that $p=\frac{\operatorname{cosec}x+1}{\cot x}$. Hence find $\cos x$ in terms of p.

[5]

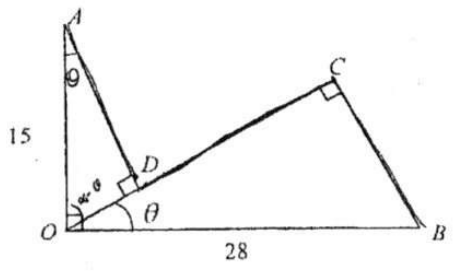
5. (i) Calculate the coordinates of the points of intersection of the graph $y=|2x-5|-3$ with the coordinates axes. [3]

(ii) Hence sketch the graph of $y=|2x-5|-3$. [2]

6. (i) Prove that $\operatorname{cosec}2\theta+\cot 2\theta=\cot\theta$ [3]

(ii) Hence, find, in radians, the angle for which $\operatorname{cosec}2\theta+\cot 2\theta=3\tan\theta$ where $0\le\theta\le\pi$. [3]

7.



The diagram shows three fixed points O, A and B such that OA = 15 cm and $OB=28$ cm and $\angle AOB=\angle ADO=\angle OCB=90°$.
The line OC makes an angle θ with the line OB, the angle θ can vary in such a way that the point D lies along the line OC. Given that $L=AD+DC+CB$,

(i) show that $L=\left(43\cos\theta+13\sin\theta\right)$ cm, [3]

(ii) express L in the form of $R\cos(\theta-\alpha)$, where R is positive and α is acute, [2]

(iii) find the value of θ for which L = 40 cm. [3]

8. The equation of the curve is $y = kx^2 + 4x + 3 + k$, where k is a constant.

(i) Find the range of values of k for which the curve lies completely above the x-axis. [4]

(ii) In the case where $k = 2$, find the values of m for which the line $y = mx - 3$ is a tangent to the curve. [4]

9. (i) Show that $x - 3$ is the factor of the cubic polynomial $2x^3 - 9x^2 + 27$. Hence factorise completely $2x^3 - 9x^2 + 27$. [3]

(ii) Express $\dfrac{(x+3)^2}{2x^3 - 9x^2 + 27}$ as the sum of 3 partial fractions. [5]

10. The function f is defined by $f(x) = 2\cos^2 x - 6\sin^2 x$.

(i) Show that $f(x)$ can be expressed as $4\cos 2x - 2$. [2]

(ii) State the minimum value of $f(x)$. [1]

(iii) State the period of $f(x)$. [1]

(iv) Sketch the graph of $y = |f(x)|$ for $0 \le x \le \pi$. Given that the number of solutions of $|f(x)| = c$ is equal to 4, find the range of values of c. [3]

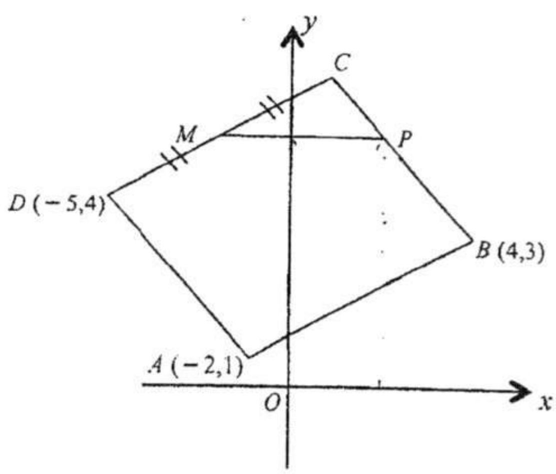
11. In the diagram, $ABCD$ is a parallelogram. The vertices A, B and D have coordinates $(-2,1)$, $(4, 3)$ and $(-5,4)$ respectively. M is the midpoint of CD. A line is drawn from M parallel to the x- axis to cut the side BC at P. Find

(i) the coordinates of C and M, [3]

(ii) the equation of the line BC, [3]

(iii) the coordinates of P, [2]

(iv) the area of $ABPMD$. [2]



12. (a) Sketch the graph of $y = x^{\frac{2}{3}}$. [2]

(b) A circle, C_1, has the equation $x^2 + y^2 - 6x + 8y - 24 = 0$.

(i) Find and the coordinates of the centre of C_1 and the radius of the circle. [3]

A second circle, C_2, has a diameter SR. The point R has coordinates $(2,-2)$ and the equation of the tangent to C_2 at S is $4y - 3x = 36$.

(ii) Find the equation of SR and hence, show that the coordinates of S is $(-4,6)$. [4]

(iii) Find the radius and the coordinates of the centre of C_2. [2]

- End of Paper -

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Marks: ______

Subject: ______ Paper No: ______ Date: ______

Name: ______ Class: ______ No: ______

For Examiner's Use

1. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

*Note: $\frac{5\pi}{12} = \frac{5}{12} \times 180°$

$= 75°$

$\frac{\pi}{12} = 15°$

$\cdot\ 75° = 45° + 30°$

$\cdot\ 15° = 45° - 30°$

$\therefore \cos\left(\frac{5\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right)$

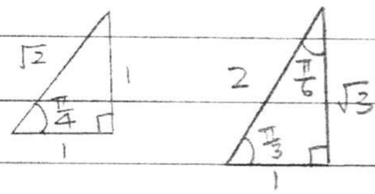
$= \left(\cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}\right) + \left(\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}\right)$

$= 2\cos\frac{\pi}{4}\cos\frac{\pi}{6}$

$= 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{\sqrt{2}}$

$= \frac{\sqrt{6}}{2}$ // (shown)



2. $\log_7 x - \log_{49}(x-2) = \log_3 1$

$\log_7 x = 0 + \log_{49}(x-2)$

$\log_7 x = \frac{\log_7 (x-2)}{\log_7 49}$

$= \frac{\log_7 (x-2)}{\log_7 7^2}$

$= \frac{\log_7 (x-2)}{2}$

$2\log_7 x = \log_7 (x-2)$

$\log_7 x^2 = \log_7 (x-2)$

$\therefore x^2 = x - 2$

$x^2 - x + 2 = 0$ //

Discriminant $= (-1)^2 - 4(1)(2)$

$= 1 - 8$

$= -7$

Since discriminant < 0, the equation has no real solutions. //

For Examiner's Use

3. Let the height of the tank be h m.

$$(1+\sqrt{2})(3-\sqrt{8})(h) = 2$$
$$(3-\sqrt{8}+3\sqrt{2}-\sqrt{16})(h) = 2$$
$$(3-2\sqrt{2}+3\sqrt{2}-4)(h) = 2$$
$$(\sqrt{2}-1)(h) = 2$$
$$h = \frac{2}{\sqrt{2}-1}$$
$$= \frac{2(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$
$$= \frac{2\sqrt{2}+2}{2-1}$$
$$= 2+2\sqrt{2}$$

$\therefore$ Height of tank is $(2+2\sqrt{2})$ m. //

4.
$$\frac{1}{p} = \frac{\operatorname{cosec} x - 1}{\cot x}$$
$$p = \frac{\cot x}{\operatorname{cosec} x - 1}$$
$$= \frac{\cot x(\operatorname{cosec} x + 1)}{(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)}$$
$$= \frac{\cot x(\operatorname{cosec} x + 1)}{\operatorname{cosec}^2 x - 1}$$
$$= \frac{\cot x \cdot (\operatorname{cosec} x + 1)}{\cot^2 x}$$
$$= \frac{\operatorname{cosec} x + 1}{\cot x} \quad \text{(proven)} \ //$$

$$\frac{1}{p} = \frac{\frac{1}{\sin x} - 1}{\frac{\cos x}{\sin x}} = \frac{1-\sin x}{\cos x}$$

$$p = \frac{\frac{1}{\sin x} + 1}{\frac{\cos x}{\sin x}} = \frac{1+\sin x}{\cos x}$$

(cont'd)

4 $\frac{1}{p} + p = \frac{1-\sin x}{\cos x} + \frac{1+\sin x}{\cos x}$

$\frac{1}{p} + \frac{p^2}{p} = \frac{(1-\sin x)+(1+\sin x)}{\cos x}$

$\frac{1+p^2}{p} = \frac{2}{\cos x}$

$\cos x(1+p^2) = 2p$

$\therefore \cos x = \frac{2p}{1+p^2}$ //

5(i) $y = |2x-5| - 3$

When $y=0$,

$0 = |2x-5| - 3$

$3 = |2x-5|$

$2x-5 = 3$ or $2x-5 = -3$

$2x = 8$ $2x = 2$

$x = 4$ $x = 1$

$\therefore$ The x-intercepts are $(4,0)$ and $(1,0)$ //

When $x=0$,

$y = |2(0)-5| - 3$

$= 5 - 3$

$= 2$

$\therefore$ The y-intercept is $(0,2)$ //

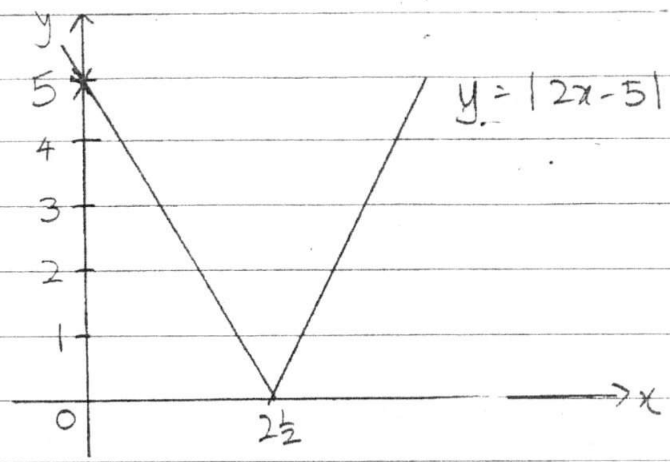
① Sketch: $y = |2x-5|$

5(ii) • When $y = 0$

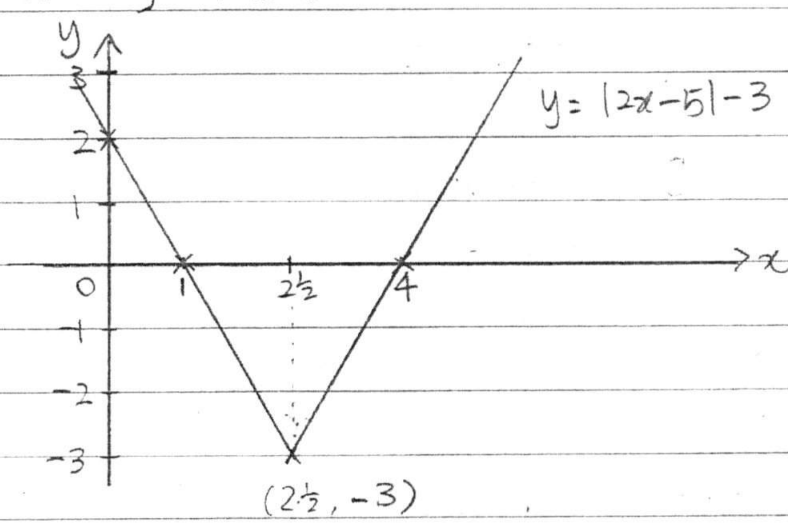
$|2x-5| = 0$

$2x - 5 = 0$

$x = 2\frac{1}{2}$



② sketch: $y = |2x-5| - 3$



This is the required sketch.

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Marks: ______

Subject: ______ Paper No: ______ Date: ______

Name: ______ Class: ______ No: ______

For Examiner's Use

6 (i) $LHS = \operatorname{cosec} 2\theta + \cot 2\theta$

$= \frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}$

$= \frac{1+\cos 2\theta}{\sin 2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta + \cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$

$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta = RHS$ (proven) //

(ii) $\operatorname{cosec} 2\theta + \cot 2\theta = 3\tan\theta$

$\cot\theta = 3\tan\theta$

$\frac{1}{\tan\theta} = 3\tan\theta$

$3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$ (θ^* in quadrants 1, 2, 3 & 4)

Basic $\angle = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}, \underbrace{\frac{7\pi}{6}, \frac{11\pi}{6}}_{(\text{reject} \because 0<\theta<\pi)}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ //

For Examiner's Use

7.(i) $\cos\theta = \frac{AD}{15}$

$AD = 15\cos\theta$ cm

$\sin\theta = \frac{OD}{15}$

$OD = 15\sin\theta$ cm

$\cos\theta = \frac{OC}{28}$

$OC = 28\cos\theta$ cm

$\therefore DC = OC - OD$

$= (28\cos\theta - 15\sin\theta)$ cm

$\sin\theta = \frac{BC}{28}$

$BC = 28\sin\theta$ cm

$L = 15\cos\theta + (28\cos\theta - 15\sin\theta) + 28\sin\theta$

$= (43\cos\theta + 13\sin\theta)$ cm (shown)//

(ii) $43\cos\theta + 13\sin\theta = R\cos(\theta - \alpha)$

$= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$= R\cos\alpha\cos\theta + R\sin\alpha\sin\theta$

$R\cos\alpha = 43$ and $R\sin\alpha = 13$

$\tan\alpha = \frac{13}{43}$

$\alpha \approx 16.8214°$

$R = \sqrt{13^2 + 43^2}$

$= \sqrt{2018}$

$\therefore L \approx \sqrt{2018}\cos(\theta - 16.8°)$//

(iii) $\sqrt{2018}\cos(\theta - 16.8214°) = 40$

$\cos(\theta - 16.8214°) = \frac{40}{\sqrt{2018}}$

Basic $\angle = \cos^{-1}\left(\frac{40}{\sqrt{2018}}\right)$

$\approx 27.072766°$

$\therefore \theta - 16.8214° = 27.072766°,\ 360° - 27.072766°$

$\theta = 43.9°,\ 349.7°$ (reject)

$\therefore \theta = 43.9°$//

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MARIS STELLA HIGH SCHOOL

Marks:

Subject: Paper No: Date:

Name: Class: No:

For Examiner's Use

8(i) $$y = kx^2 + 4x + (3+k)$$

For curve to lie completely above x-axis, discriminant < 0

$$(4)^2 - 4(k)(3+k) < 0$$

$$16 - 12k - 4k^2 < 0$$

$$k^2 + 3k - 4 > 0$$

$$(k-1)(k+4) > 0.$$

$\therefore$ $k < -4$ or $k > 1$. $\rightarrow \because k > 0, \therefore k > 1$ //

(ii) $$y = 2x^2 + 4x + 5 \quad \text{— ①}$$

$$y = mx - 3 \quad \text{— ②}$$

Sub ② into ①,

$$mx - 3 = 2x^2 + 4x + 5$$

$$2x^2 + 4x - mx + 5 + 3 = 0$$

$$2x^2 + (4-m)x + 8 = 0$$

For line to be tangent to curve, discriminant $= 0$.

$$(4-m)^2 - 4(2)(8) = 0$$

$$16 - 8m + m^2 - 64 = 0$$

$$m^2 - 8m - 48 = 0$$

$$(m+4)(m-12) = 0$$

$\therefore$ $m = -4$ or 12 //

For Examiner's Use

9(i) Let $f(x) = 2x^3 - 9x^2 + 27$

$f(3) = 2(3)^3 - 9(3)^2 + 27$

$= 54 - 81 + 27$

$= 0$

$\therefore (x-3)$ is a factor of $f(x)$. (Shown)//

$f(x) = (x-3)(2x^2 + bx - 9)$

comparing coefficient of x^2,

$-9 = -6 + b$

$b = -3$

$\therefore f(x) = (x-3)(2x^2 - 3x - 9)$

$= (x-3)(2x+3)(x-3)$

$= (x-3)^2(2x+3)$//

(ii) $\dfrac{(x+3)^2}{2x^3 - 9x^2 + 27} = \dfrac{(x+3)^2}{(x-3)^2(2x+3)}$

Let $\dfrac{(x+3)^2}{(x-3)^2(2x+3)} = \dfrac{A}{x-3} + \dfrac{B}{(x-3)^2} + \dfrac{C}{2x+3}$

$(x+3)^2 = A(x-3)(2x+3) + B(2x+3) + C(x-3)^2$

When $x = 3$,

$(3+3)^2 = 0 + B[2(3)+3] + 0$

$36 = 9B$

$B = 4$

When $x = -\frac{3}{2}$

$(-\frac{3}{2}+3)^2 = 0 + 0 + C[(-\frac{3}{2}) - 3]^2$

$\frac{9}{4} = \frac{81}{4}C$

$C = \frac{1}{9}$

$\longrightarrow$

(Cont'd)

9(ii) When $x=0$,

$$(0+3)^2 = A(-3)(3) + 4(3) + \frac{1}{9}(-3)^2$$

$$9 = -9A + 12 + 1$$

$$-4 = -9A$$

$$A = \frac{4}{9}$$

$$\therefore \frac{(x+3)^2}{2x^3-9x^2+27} = \frac{4}{9(x-3)} + \frac{4}{(x-3)^2} + \frac{1}{9(2x+3)} \quad //$$

10(i)

$$2\cos^2 x - 6\sin^2 x = 2\cos^2 x - 6(1-\cos^2 x)$$

$$= 2\cos^2 x - 6 + 6\cos^2 x$$

$$= 8\cos^2 x - 6$$

$$= 4(2\cos^2 x) - 6$$

$$= 4(\cos 2x + 1) - 6$$

$$= 4\cos 2x + 4 - 6$$

$$= 4\cos 2x - 2 \quad \text{(shown)} //$$

(ii)

$$-1 \le \cos 2x \le 1$$

$$-4 \le 4\cos 2x \le 4$$

$$-6 \le 4\cos 2x - 2 \le 2$$

$\therefore$ Minimum value of $f(x)$ is -6 //

(iii)

$$\text{period} = \frac{2\pi}{2}$$

$$= \pi \quad //$$

$\longrightarrow$

(cont'd) 10(iv)

$y = 4\cos 2x - 2, \quad 0 \le x \le \pi$

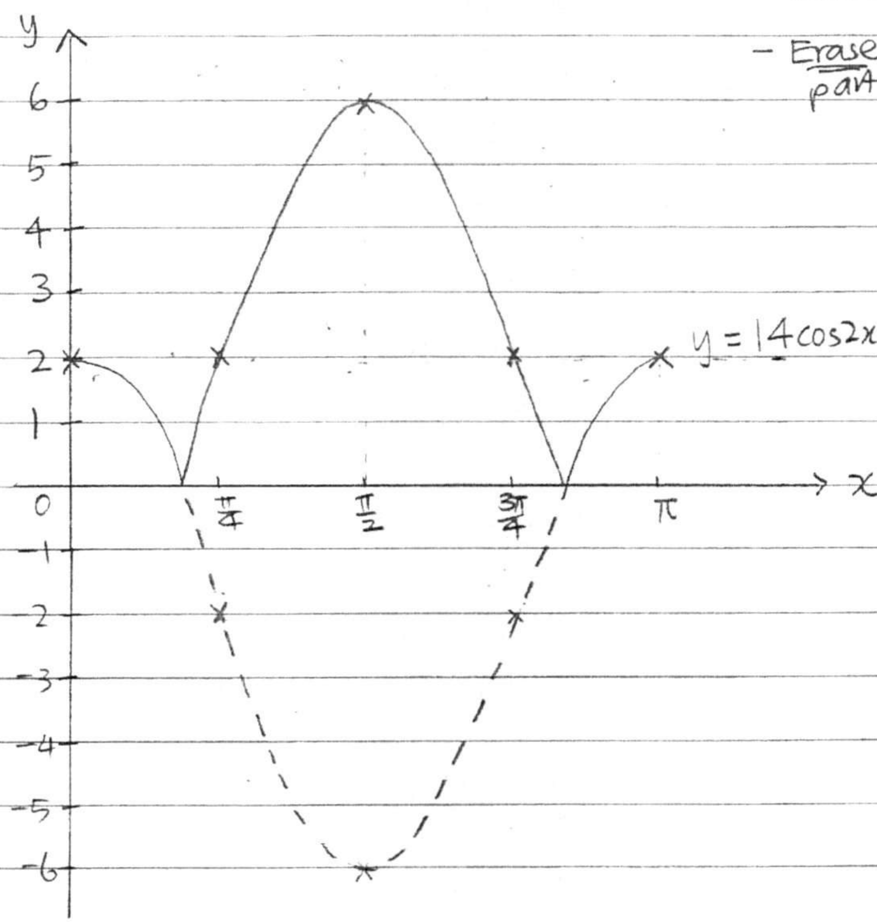
For Examiner's Use

Amplitude $= 4$

Period $= \pi$

Range: $-6 \le f(x) \le 2$

*Recall:
- Draw $y = 4\cos 2x - 2$ first.
- Then draw $|f(x)|$.
- Erase unwanted part of graph.



Note: left as a guide. To erase after drawing $|f(x)|$

To obtain 4 solutions, the line $y = c$ must cut the $|f(x)|$ graph at 4 points.

$\therefore \quad 0 < c \le 2$ //

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MARIS STELLA HIGH SCHOOL

Marks:

Subject: Paper No: Date:

Name: Class: No:

For Examiner's Use

1 (i) Let coordinates of C be (e, f).

midpoint of AC = midpoint of BD

$$\left(\frac{-2+e}{2}, \frac{1+f}{2}\right) = \left(\frac{4+(-5)}{2}, \frac{3+4}{2}\right)$$

$\therefore \frac{-2+e}{2} = \frac{4+(-5)}{2}$	$\frac{1+f}{2} = \frac{3+4}{2}$
$-2+e = -1$	$1+f = 7$
$e = 1$	$f = 6$

$\therefore$ coordinates of C are $(1, 6)$. //

Midpoint of CD, M $= \left(\frac{-5+1}{2}, \frac{4+6}{2}\right)$

$= (-2, 5)$

$\therefore$ coordinates of M are $(-2, 5)$ //

(ii) Gradient of BC $= \frac{6-3}{1-4}$

$= -1$

$$\frac{y-3}{x-4} = -1$$

$$y-3 = -(x-4)$$

$$y = -x+4+3$$

$$\therefore y = -x+7$$

Equation of BC is $y = -x+7$ //

For Examiner's Use

(Cont'd)

11 (iii) Since MP is // to x-axis, equation of line MP is $y = 5$.

Sub $y = 5$ in $y = -x + 7$,

$$5 = -x + 7$$

$$x = 2$$

$\therefore$ coordinates of P are $(2, 5)$. //

(iv) $$\text{Area } ABPMD = \frac{1}{2}\begin{vmatrix} -2 & 4 & 2 & -2 & -5 & -2 \\ 1 & 3 & 5 & 5 & 4 & 1 \end{vmatrix}$$

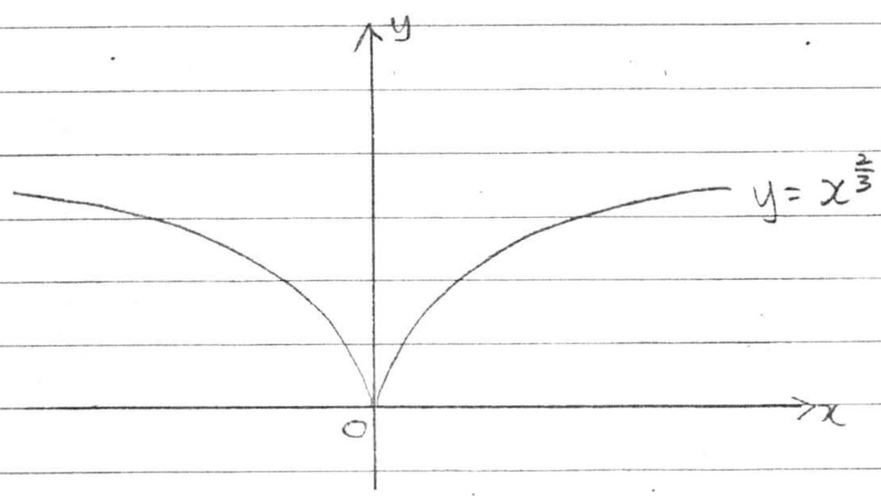
$$= \frac{1}{2}\left[(-6)+20+10+(-8)+(-5)-4-6-(-10)-(-25)-(-8)\right]$$

$$= \frac{1}{2}[11 + 33]$$

$$= 22 \text{ units}^2$$ //

12(a) $$y = x^{\frac{2}{3}}$$

$$= \sqrt[3]{x^2}$$



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MARIS STELLA HIGH SCHOOL

Marks:

Subject: Paper No: Date:

Name: Class: No:

For Examiner's Use

12(bi)

$$x^2+y^2-6x+8y-24=0$$

$$x^2-6x+\left(-\tfrac{6}{2}\right)^2-\left(-\tfrac{6}{2}\right)^2+y^2+8y+\left(\tfrac{8}{2}\right)^2-\left(\tfrac{8}{2}\right)^2-24=0$$

$$(x-3)^2-9+(y+4)^2-16-24=0$$

$$(x-3)^2+(y+4)^2-49=0$$

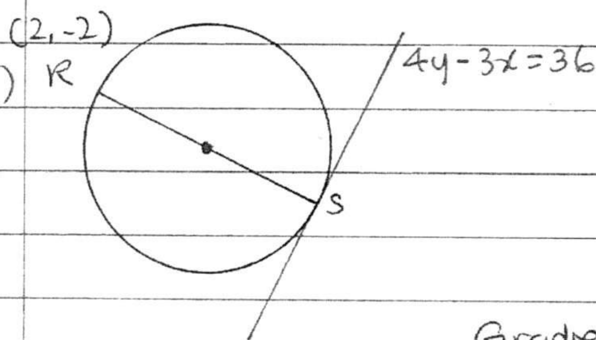
$$(x-3)^2+(y+4)^2=49$$

$\therefore$ coordinates of centre of C_1 are $(3,-4)$.

Radius of $C_1 = \sqrt{49}$

$= 7$ units

(ii)



$$4y-3x=36$$

$$4y=3x+36$$

$$y=\tfrac{3}{4}x+9$$

Gradient of tangent at S $= \frac{3}{4}$

Gradient of RS $= -\left(\frac{1}{\frac{3}{4}}\right)$

$= -\frac{4}{3}$

$\Big\}$ *Recall: tan $\perp$ rad.

$$\frac{y-(-2)}{x-2}=-\frac{4}{3}$$

$$y+2=-\tfrac{4}{3}(x-2)$$

$$y=-\tfrac{4}{3}x+\tfrac{8}{3}-2$$

$$y=-\tfrac{4}{3}x+\tfrac{2}{3}$$

$\therefore$ Equation of RS is $y=-\frac{4}{3}x+\frac{2}{3}$.

$\longrightarrow$

(cont'd)

12b(ii)

$$y = \frac{3}{4}x + 9 \quad — (1)$$
$$y = -\frac{4}{3}x + \frac{2}{3} \quad — (2)$$

Sub. (1) into (2),

$$\frac{3}{4}x + 9 = -\frac{4}{3}x + \frac{2}{3}$$
$$\frac{25}{12}x = -\frac{25}{3}$$
$$x = -4$$

Sub $x = -4$ into (1),

$$y = \frac{3}{4}(-4) + 9$$
$$y = -3 + 9$$
$$\therefore y = 6$$

Hence, coordinates of S are $(-4, 6)$. (shown)

(iii) centre of C_2 = midpoint of RS

$$= \left(\frac{2 + (-4)}{2}, \frac{-2 + 6}{2}\right)$$
$$= (-1, 2)$$

$\therefore$ coordinates of centre of C_2 are $(-1, 2)$

Radius of $C_2 = \sqrt{(-1-2)^2 + [2-(-2)]^2}$
$= \sqrt{25}$
$= 5$ units

For Examiner's Use